

THERMODYNAMICS FORMULATION OF ECONOMICS

Session: Theoretical Advances II

The International Conference on Thermodynamics 2.0

Burin

Gumjudpai

บุรินทร์ กำจัดภัย

June 23, 2020

The Institute for Fundamental Study “The Tah Poe Academia Institute”, Naresuan University
Phitsanulok, Thailand

KEY FEATURES

- New ideas of thermodynamics (truly endogenous effect | effect structure diagram)
- Suggesting that econ. “stock” variables as thermo. “coordinates” and thermo. “potential”
- Economics analog of “mechanical pair of variables” and “thermal pair of variables”
: reasonable (thermo+effect structure)-motivated EoS for Economics, solid b/g for statistical mechanics modeling
- Econometrics EoS modelling motivated by this work:
alternative concept in defining coefficient in an econometric model
- Zero, First, Second and Third Laws
- Comment of invalid concept of Consumer Surplus
- In Econ. Maximization of Utility needs at least 2 commodities (I.C. curve) with limited budget constraint (constraint optimization)
- In this work: Maximization of **Generalized Utility/(average personal wealth)** max. with the 2nd Law
- Concept of **Total Wealth** and Generalized Utility and **Generalized Surplus**

THERMODYNAMICS OF INTEGRATION (TOI) STATEMENT

INTEGRATION OF THE NATURAL AND SOCIAL SCIENCES

- ❖ Economics and Thermodynamics: Existence of Equilibrium States
- ❖ Both describe aggregated phenomena.
- ❖ Connection not trendy
(most works on economics focus more on topics such as game theory, pheno. forecasting rather than b/g theory).
- ❖ Analogies should open new perspectives of both subjects.
- New Effect Structure Rules and Diagrams: new add-on modification
- EoS in Economics: more theoretical-solid econometric modeling
- ❖ Hope: complete picture of “thermodynamical paradigm” of economics.

INTRODUCTION

- ❖ Physics 2 Types: EoM and EoS
- ❖ **EoM is from symmetries** (e.g. canonical transformation etc.)
- ❖ **EoS is from empirical axiom – Aggregated Result**
- ❖ EoS is 2 Dim constraint surface embedded in 3-Dim “Home” (space)
- ❖ Variable’s roles of being the cause and being affected
- ❖ Coordinates: X (extensive), Y (intensive) T (temperature)
- ❖ Products of X and Y must give work term in the first law of thermodynamics

$$dU = \delta W + \delta Q + \mu dn$$

with $\delta W = YdX$ and no μdn term for a closed system (considered here)

INTRODUCTION

❖ Mechanical Pairs:

1. **extensive coordinate from mechanical definition, (X):**
Examples are volume (V), magnetization ($\mathcal{M}$), electrical polarization ($\mathcal{P}$)
2. **intensive coordinate from mechanical definition, (Y):**
Examples are (negative value of) pressure ($-P$), magnetic field intensity (B_0)
electric field intensity (E),

❖ Work Term:

$$\delta W = \sum_i Y_i dX_i \quad \text{e.g.} \quad \delta W = -PdV$$

❖ Heat Term:

$$\delta Q = TdS.$$

INTRODUCTION

- ❖ Mechanical Pair (Y_i, X_i) with number of pair $i = 1 \dots n$
- ❖ From $2(n+1)$ tuples (X_i, Y_i, S, T) with quasi-static process or closed to equilibrium approx., $dS = 0$ this become $(2n+1)$ tuples

$$\mathfrak{M} = (X_i, Y_i, T)$$

Manifold space (given by nature: empirical laws + assumption of existing of equilibrium state)

This is “**empirical axioms**”

- ❖ Example is $\mathfrak{M} = (V, P, T)$ for $n = 1$

INTRODUCTION

- ❖ incomplete meaning of ideal gas laws (no description of cause and effect)

$$V \propto T \quad V \propto 1/P \quad P \propto T$$

- ❖ These three empirical facts (constant n) form EoS relating 3 coordinates (2 d.o.f. left)

$$PV = nRT \quad (\text{ideal gas}) \quad \text{or in general hydrostatics system} \quad g(V, P, T) = 0$$

Hydrostatic EoS is 2-Dim constraint surface in the home $\mathfrak{M} = (V, P, T)$ 3-Dim space
a concept of empirical axioms.

V , P and T are “coordinates of thermodynamics space” (S fixed)

INTRODUCTION

❖ Thermodynamics Potential “field” $U = U(P, V, T)$

Constrained with EoS hence $U = U(V, T) = U(P, T) = U(V, P)$

In “natural coordinate” (suggested by thermo. Postulate – 1st law), $U = U(S, V)$ (closed system)

$U(S, V) = TS - PV$ (constrained with EoS surface, left with 2 variables)

Existence of EoS “equiv. to” Existence of Potential

$$H = U + PV \quad \mathcal{F} = U - TS \quad G = \mathcal{F} + PV = U - TS + PV$$

with Legendre transformation

❖ Distinction of Thermodynamics Potentials | Coordinate Space Variables | Energy Transfer

❖ In economics: “stock quantity” or “flow quantity”.

Flow: work and heat.

Stock: *which stock variables should be potential / should be coordinates?*

INTRODUCTION

- ❖ No concept of EoS in Econ., and as a result, no concept of potential in Econ
(Debrue 1972, Mas-Colell 1985)

“Economics, because it does not recognize an equation of state or define prices intrinsically in terms of equilibrium, lacks the close relation between measurement and theory physical thermodynamics enjoys.”

(Smith and Foley 2008)

- ❖ Similarities of Thermodynamics and Economics:

1. Equilibrium determined by a set of pair of dual state variables
2. Constraint of conservation of a quantity in a closed system (not necessary isolated)
3. Preference of a quantity not to decrease in approaching equilibrium state

Smith and Foley 2008 proposed $g(Q, Pr, MRS) = 0$ as EoS.
(based on initial setting of variable maximization: Utility as Entropy: Two Goods)

ENDOGENOUS, EXOGENOUS VARIABLES IN ECONOMICS

For real-value set of variables $\{x_1, \dots, x_n\}$

If there exists $x_1 = f(x_2, \dots, x_n)$

and the assumed model has just only x_2 as the only dependent var. (according to not-yet-to-be known knowledge)

$$x_1 = f(x_2, a_3, a_4, \dots, a_n) = f_1(x_2)$$

x_2 is said to be endogenous variable (it is included in the “model” of consideration)

Any changes in the other x_n (except x_2) are exogenous effect. (because they are not included in the assumed model)

However, here we have new idea motivated from EoS to identify which should be endogenous / exogenous variables.

DISTINCTIONS BETWEEN PHYSICS AND ECONOMICS

Physics: [how things change]

- Importance of having manifold as home of either EoM or EoS (Thermodynamics)
- Prediction power
- Unification picture.
- Short Relaxation Time
- Controlled conditions
- Coordinates and Potential (or Field)
- Richer in fundamental units (6)
(meter | second | kg | Coulomb | Kelvin | mole)

Traditional Economics: [making choices to allocate limited resources]

- No EoS (so far)
- Importance of empirical formulae with prediction power
- Unification incomplete: separated by “schools of thoughts”
- Long Relaxation Time/hardly attain equilibrium state
- Frequently minor disturbed or big shock
- Potential (or Field) and Coordinates
(different characters of coordinates to EoM or EoS)
- Fewer in fundamental units (4)
(money unit | time unit | # of Goods | # of Agents (Consumer, Seller, Company))

PHYSICS AGENTS | SOCIAL AGENTS

- Name (Phys and Social)
- Properties (Phys) | Personal Status Elasticity (Social)
- Duty (How Law tells its Properties (Phys)) | Social Duty due to Personal Status Elasticity (Social)
- Systematic Uncertainty (Phys) | Systematic Risk (Social)
- Choice or Option (only Social) due to many factors such as Preference or Memory

EOM, EOS AND CARATHÉODORY'S AXIOMS

- EoM: rational reasoning - deduction
- EoS: empirical axioms – deduction (Carathéodory's axioms) with prediction power
- Carathéodory's axioms: having a set of infinite number of states:
 $s = s(X, Y)$ (entropic state) or $t = t(X, Y)$ (thermal state)
together with postulates on conservation of internal energy and second law to be deducted
- Analogous pictures to Carathéodory's axioms can be set for any mechanical pair X, Y
- This is unlike conventional thermodynamics:
 Induction of EoS with
 4 Laws postulated **to be deducted**.

PREVIOUS LITERATURES

- Fisher (1892) – Thermodynamics analogy to grad. of Utility w.r.t. quant. of goods
: grad. of Internal Energy (mismatching!)
- Walras (1909) – force of potential energy analogy to grad. of Utility w.r.t. quant. of goods
- Abandoned for many years until Samuelson's critique (1960)

“The formal mathematical analogy between classical thermodynamics and mathematic economic systems has now been explored. This does not warrant the commonly met attempt to find more exact analogies of physical magnitudes - such as entropy or energy - in the economic realm. Why should there be laws like the first or second laws of thermodynamics holding in the economic realm? Why should 'utility' be literally identified with entropy, energy, or anything else? Why should a failure to make such a successful identification lead anyone to overlook or deny the mathematical isomorphism that does exist between minimum systems that arise in different disciplines”.

PREVIOUS LITERATURES

- **Lisman (1949)** (Utility | S)

$Pr \cdot Q$ as analogous to the product of pressure and volume $P \cdot V$ of the ideal gas EoS

{should be X and Y rather than directly matching to V, P : need to check with effect structure}

- **Saslow (1999)**: (Utility | $-U$), Price to chemical potential etc.

{Utility should be related to S | chemical potential should be used for open system: not for a closed system}

- **Smith and Foley (2008)**: Conservation of commodities, Utility as entropy with EoS as

$$g(Q, Pr, MRS) = 0.$$

Marginal rate of substitution (MRS) plays a role of temperature: this is due to the analog problem of “constrained optimization” which takes demand quantities (Q_j^d) of each type of commodity as space coordinates.

{Our proposal here: “Temp. is concentration of internal energy/mole (for a closed, perfect system).”
Temp is related to “potential variable” in form of the 1st law. MRS does not have this character and it is of a system with more than one commodity}

EXTENSIVE CO., INTENSIVE CO., FIRST LAW

mechanical pair.

extensive coordinate from mechanical definition, (X)

size-dependent

intensive coordinate from mechanical definition, (Y)

size-independent

$$dU = \delta W + \delta Q + \cancel{\mu dn}$$

0 for closed system

heat term $\delta Q = TdS$

work term, $\delta W = YdX$

1st Law (conservation)

$\left. \begin{array}{c} X \\ T \\ Y \\ S \end{array} \right\}$ coordinates

internal energy $U = U(S, X_i, n)$.

Field or Potential

EoS

$$g_i(X_i, Y_i, T) = 0$$

constraint

NOTICE THAT IN AN EOS:

Lacks of representing some correct physical senses: e.g. $PV = nRT$ or generally $g(V, P, T) = 0$

(A) “ P can affect T or V directly” [initial change in P is from externality]

(B) “ T can affect P directly” [initial change in T is from externality]

“ T can not affect V directly, but only via P ” [initial change in T is from externality]

“ V can affect neither T nor P ” [change in V is not allowed from externality but only from P , hence V is passive]

If T is changed by external effect, then T internally affects P to change.
As a result, change in P internally affects V to change

Effects from externality is dubbed “**exogenous**” | Only T and P can take the exogenous effect.

Functions related “internal” EoS variables (P, V, T) is “**endogenous**”.

Some of these functions represent the effects in (A) and (B) are “truly endogenous functions”

TRULY ENDOGENOUS FUNCTION

Consider $V = V(P, T)$ (seems fine) however in fact it is $V = V \circ P(T) = V(P(T))$

A mapping with **non-composite function** from “cause” variable to “effect” variable

Hence **truly endogenous function** are, for examples,

$$V = \tilde{V}(P)$$

$$P = \tilde{P}(T)$$

$$T = \tilde{T}(P)$$

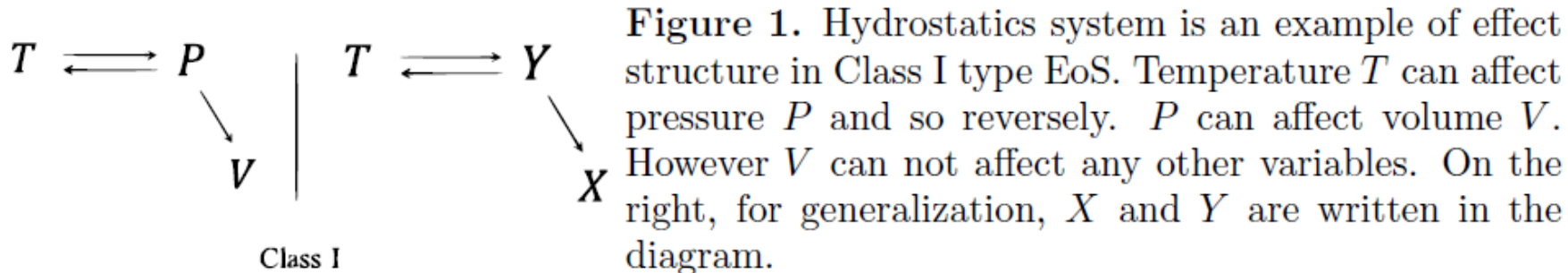
In which **argument of function in the cause** and **the value of function is the effect**.

This is performed with **directed graph**.

DIAGRAM OF EFFECT STRUCTURE: CLASS I

Class I: (two opposite-direction arrows linking T and Y)

Class I: $V = \frac{nRT}{P} \quad \Rightarrow \quad X = k \left(\frac{T}{Y} \right)$



Arrows represent **truly endogenous function** and they represent direction of cause and effect. Initial change in T or Y are possible from both exogenous effect and endogenous effect.

DIAGRAM OF EFFECT STRUCTURE: CLASS II

Class II: (two opposite-direction arrows linking T and X)

$$\text{Class II:} \quad \mathcal{M} = C \frac{B_0}{T} \quad \Rightarrow \quad X = k \left(\frac{Y}{T} \right)$$

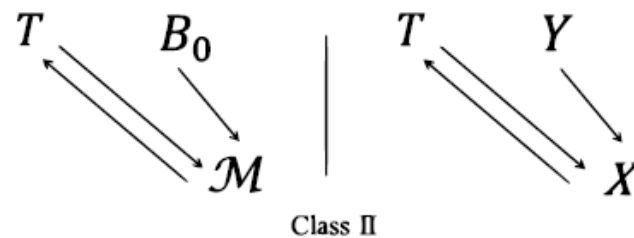


Figure 2. Paramagnetic system is an example of effect structure in Class II type EoS. Temperature T can affect magnetization $\mathcal{M}$ and so reversely. External magnetic field intensity B_0 can affect $\mathcal{M}$. However B_0 can not be affected by other variables.

CRITERIA RULES FOR EOS STATUS OF EMPIRICAL LAWS — A PROPOSAL OF FORMAL PROCEDURE

Aspects noticed by our study and shall be applied as criteria of the status of an EoS, $g(X, Y, T) = 0$ of a system. These come as some novel rules for an empirical relation to be considered as an EoS. These rules are

1. Number of the arrows is three.
2. There is at least one arrow pointing $Y \rightarrow X$.
3. Only T and(or) Y (apart from causing truly endogenous effects) can also be considered as exogenous influence or a shock¹.

¹ One might argue that X (e.g. volume) can take other values in the equation that related Y and T , i.e. $Y = Y(T(X)) = Y \circ T(X)$ or $T = T(Y(X)) = T \circ Y(X)$ so that changing X might look like exogenous effect. However for a system in reality, given a value of X at beginning, one X can not change value of X unless with endogenous effect from Y and T .

3-DIM SPACE OF THERMO. ECONOMICS

Observation empirical under some assumption (just as when considering the “ideal” gas case), a market is “perfect competitive” with “information symmetric” at equilibrium.

Proposing “**-Price**” as intensive coordinate $-Pr$ with unit $\frac{\text{Money Unit}}{\text{Unit Quantity of Good}}$

Proposing “**Demand Quantity**” as extensive coordinate Q^d with unit Quantity of Good

Proposing “**Average personal Wealth**” as temperature $\varphi = \frac{W}{N}$ with unit $\frac{\text{Money Unit}}{\text{Unit Quantity of Consumer}}$

The EoS $g(Q^d, Pr, \varphi) = 0$

3-DIM SPACE OF THERMO. ECONOMICS

Keys:

- “**Price**” Pr as intensive coordinate: the influence/unit of good as $P = F/A$ or $|\vec{E}| = F_e/q$ or magnetic field intensity
- “**Demand Quantity**” Q^d as extensive coordinate: “aggregated and extensive” as volume or magnetization or electric polarization
- “**Average Personal Wealth**” $\varphi = \mathcal{W}/N$ as analogous of (ideal gas) temperature

$$T = 2U/(3Nk_B) = 2U/(3nR)$$

3-DIM SPACE OF THERMO. ECONOMICS

- **Total Wealth function** $\mathcal{W} = \text{wealth in happiness} + \text{wealth in asset}$

$$\mathcal{W} = \mathcal{W}(Q^d, Pr, \varphi)$$

is a potential (field) function of (Q^d, Pr, φ) . It plays a role of internal energy.

- **Generalized Utility function** (heat term) $\delta \mathcal{Q}_{\text{util}}^d = \varphi dS^d$

$$\delta \mathcal{Q}_{\text{util}}^d = \delta(\text{pleasure or opportunity of ownership in assets}) + \delta(\text{happiness of utilizing commodity})$$

unlike microeconomics, maximizing of $\delta \mathcal{Q}_{\text{util}}^d$ is not derived by optimizing wr.t. Q_j^d (coordinates) under [a budget constraint](#). Instead **generalized utility is maximized with 2nd law**. This is achieved even for one commodity consumption from expenditure or even without expenditure $\delta W^d = -Pr dQ^d$, but only

With $dS^d \neq 0$. The change $\Delta S^d = \int \delta \mathcal{Q}_{\text{util}}^d / \varphi$ is change in generalized utility per unit of average personal wealth.

THERMAL STATE OF THERMO. ECONOMICS

- Following Carathéodory's axioms, observations suggest existence of a set of infinite “thermal” states

$$f(Pr_1, Q_1^d) = \varphi_1 \quad \text{and} \quad f(Pr_2, Q_2^d) = \varphi_2$$

and so on...

- Mechanical changes in total wealth of consumers is defined by $\delta W^d = -Pr dQ^d$
work term (demand-side) of “economics version of mechanical pair”, $(-Pr, Q^d)$

This is the expenditure of the process.

- Economics version of **adiabatic process** (heat = 0), $d\mathcal{W} = \delta W^d = -Pr dQ^d$

Idealistically this is a **spending without gaining any utility**.

This is a **strictly 1st law** according to Carathéodory's axioms.

ENTROPIC STATE OF THERMO. ECONOMICS

- Following Carathéodory's axioms, observations suggest existence of a set of infinite “entropic” states

$$s(Pr_1, Q_1^d) = S_1^d \quad \text{and} \quad s(Pr_2, Q_2^d) = S_2^d$$

and so on...

- **Thermal changes** in total wealth of consumers is defined by heat term (demand-side) of “economics version of thermal pair”, (φ, S^d)
 $\delta \mathcal{Q}_{\text{util}}^d = \varphi dS^d$ (reversible) is the generalize utility (heat) gained from spending δW^d
- For an economics version of **isovolumic process** (work = 0),

$$d\mathcal{W} = \delta \mathcal{Q}_{\text{util}}^d = \varphi dS^d$$

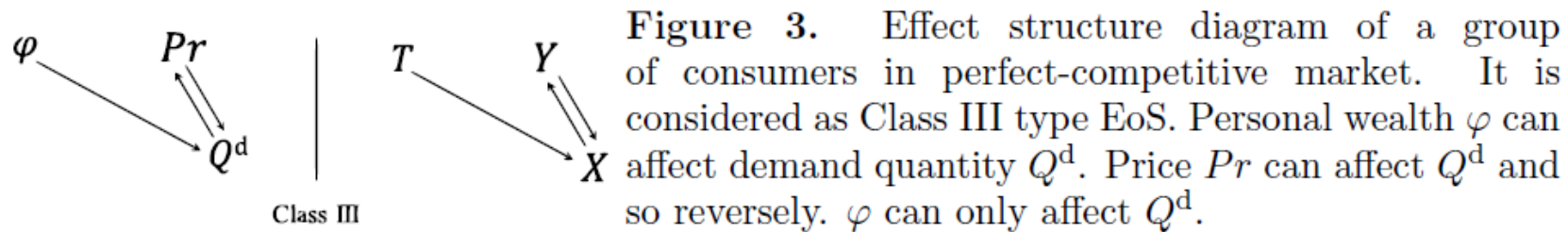
Idealistically this is a **gaining of utility without spending**.

EFFECT STRUCTURE OBEYING

- Why are we assured of the space (Q^d, Pr, φ) ?

Following “The Effect Structure Proposal” we may write and sketch

$$Q^d = kf(Pr, \varphi) \quad \text{or for generalization,} \quad X = kf(Y, T)$$



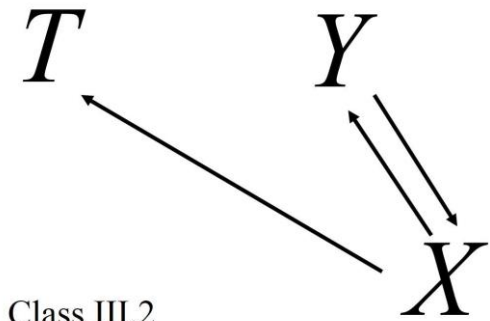
- This obeys the “Criteria Rules” suggesting a Class III
Class III: (two opposite-direction arrows linking Y and X)

Above is Class 3.1.

Class 3.2 ($X \rightarrow T$) | Class 3.3 ($T \rightarrow Y$) | Class 3.4 ($Y \rightarrow T$) are also allowed:

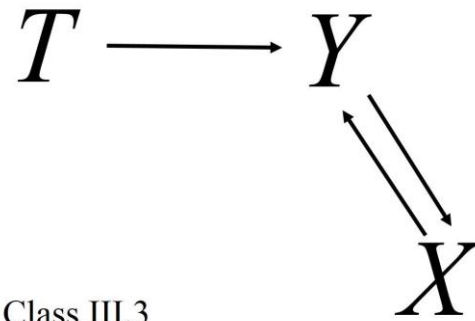
EFFECT STRUCTURE OBEYING

Class 3.2



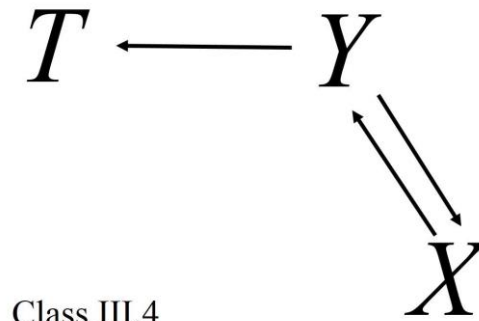
Class III.2

Class 3.3



Class III.3

Class 3.4



Class III.4

THERMAL CONTACT (WEALTH CONTACT)

Two consumers with different personal wealth: φ_1, φ_2

Can share by: marriage, partners, adoption, family or being together as a household

At equilibrium, each has equal personal wealth (under some assumptions).

What is to be transferred?

Answer: Generalized Utility (assets ownership + happiness) all in money unit. **Assets ownership can be transferred with or without expenditure.** For **thermal contact**, it is without expenditure (work term).

Forms of Gen. Utility differ for types of commodity or assets and the way one consume, e.g. for food, utility is likely to be happiness of eating; for furniture, utility is ownership and happiness of using them; for **money**, utility is **ownership** and **opportunity of investment or spending**.

ISOTHERMAL PROCESS

Group of consumers with one equally constant average personal wealth: φ

Processes are governed by Smith's price mechanism: law of demand
(microeconomics is a specific case of our theory)

ISOBARIC PROCESS

One single type of commodity: with one constant price

Demand quantity Q^d depends on how rich (in average) φ the consumers are.
Richer people can buy for eating or using more?

0TH LAW

Existing of average personal wealth φ with personal wealth equilibrium (slowly approaching)

1ST LAW

Existing of wealth function $\mathcal{W} = \mathcal{W}(Q^d, Pr, \varphi)$ with conservation:

$$d\mathcal{W} = \delta \mathcal{Q}_{\text{util}}^d + \delta W^d$$

2ND LAW

Existing of **entropic function** S^d interpreted as **generalized utility/(unit of average personal wealth)**

$$\Delta S^d \geq \int (\delta \mathcal{Q}_{\text{util}}^d / \varphi)$$

In any processes, utility/(unit of personal wealth) does not decrease.

Understanding meaning of S^d as Gen. Utility/(unit of personal wealth):

“Rich people (high personal wealth) have less happiness in using product.
They tend to look over the value of things,
because it is easy for them to buy the products (service) again whenever they want.
This is opposite to the poor.”

3RD LAW

At zero personal wealth state, Gen. Utility/unit of personal wealth is zero.
However, it takes infinite steps to reach the zero personal wealth state.

“Since in our theory, wealth includes happiness, hence there is no way to take it away completely.”

“If zero personal wealth state exists (no assets and no even happiness),
hence there is zero utility for him/her. (no happiness of using, no ownership)”

(very similar to Buddhism Philosophy)

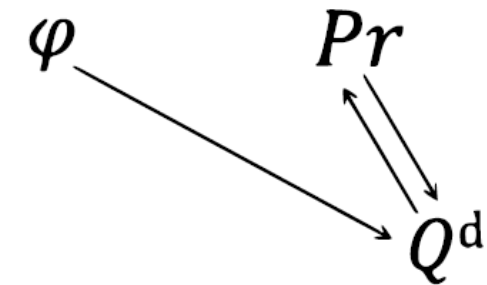
MODELLING OF EOS: DEMAND FUNCTION

- Coordinates (Q^d, Pr, φ)
- Potential or Field $\mathcal{W} = \mathcal{W}(Q^d, Pr, \varphi)$ with natural coordinates (from first law) S^d and Q^d (with EoS, $g(Q^d, Pr, \varphi) = 0$ hence independent coordinates reduce to 2 d.o.f.
- Truly Endogenous Functions

$$Q^d = \tilde{Q}^d(Pr) \quad Pr = Pr(\tilde{Q}^d)$$

$$Q^d = \tilde{Q}^d(\varphi)$$

- No truly endogenous function between Pr and φ
- Effect structure is described by diagram (**directed graph**)
- Each arrow must be Truly Endogenous Function



MODELLING OF EOS: DEMAND FUNCTION

- Real Gas with Virial Expansion

$$\frac{PV}{nRT} = Z(v) = 1 + \frac{B}{v} + \frac{C}{v^2} + \frac{D}{v^3} + \frac{E}{v^4} + \dots$$

- Application to demand-side Econ EoS, (approx. at 1st order)

$$\frac{Pr q^d}{\alpha \varphi} = Z(q^d) \simeq 1 + \frac{B}{q^d} \quad \text{whereas } q^d \equiv Q^d/N \text{ is average personal demand.}$$

Taking logarithm, $\ln(q^d) \simeq \ln \alpha + \ln \varphi - \ln Pr + \ln \left(1 + \frac{B}{q^d} \right)$

Realistic aspect of data (property) is to be decoded from B —

MODELLING OF DEMAND FUNCTION EOS

- Modelling with total differential (Simple Solid analogy)

$$q(Q^d, Pr, T) = 0. \quad \longrightarrow \quad dQ^d = \left(\frac{\partial Q^d}{\partial \varphi} \right)_{Pr} d\varphi + \left(\frac{\partial Q^d}{\partial Pr} \right)_{\varphi} dPr$$

$$\beta_{Pr} \equiv \frac{1}{Q_0^d} \left(\frac{\partial Q^d}{\partial \varphi} \right)_{Pr} = \frac{E_{\varphi}^d}{\varphi_0}$$

$$\kappa_{\varphi} \equiv -\frac{1}{Q_0^d} \left(\frac{\partial Q^d}{\partial Pr} \right)_{\varphi} = -\frac{E_{Pr}^d}{Pr_0}$$

Elasticities could depend on other factors as similar to the factors in EoS of simple solid.

$$Q^d(Pr, \varphi) = Q_0^d [1 + \beta_{Pr}(\varphi - \varphi_0) - \kappa_{\varphi}(Pr - Pr_0)] \quad \longleftarrow \text{EoS}$$

$$Y = 1 + \beta_{Pr}X_1 - \kappa_{\varphi}X_2 + u \quad \text{EoS in form of regression equation} \quad u \text{ is an error term.}$$

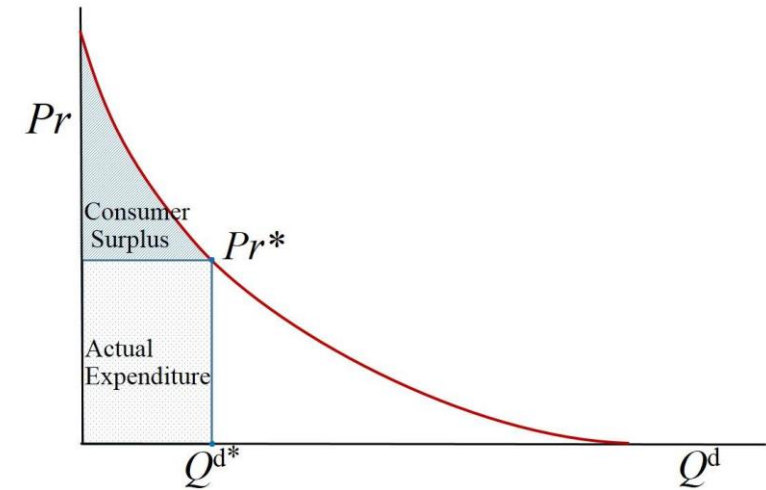
$$\begin{aligned} Y &\equiv \frac{Q^d}{Q_0^d} \\ X_1 &\equiv \varphi - \varphi_0 \\ X_2 &\equiv Pr - Pr_0 \end{aligned}$$

One can use econometric methods for e.g. cross-section or time series data with the EoS

CONSUMER SURPLUS

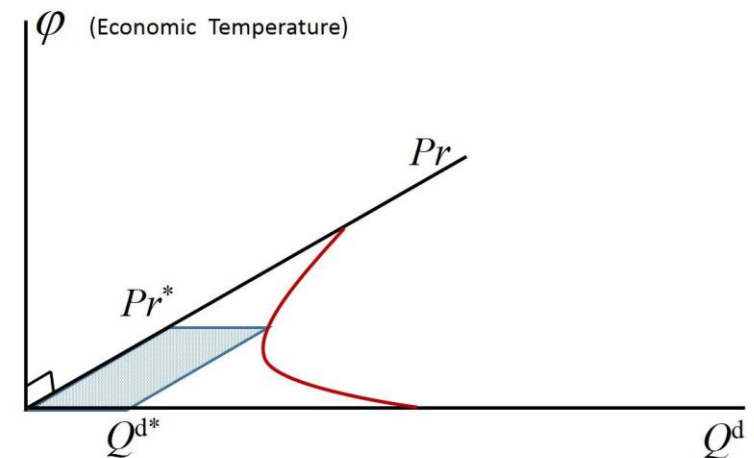
Concept of consumer surplus exists in economics without realization of φ coordinate, the surplus is calculated from the area in picture, which based on concept of 2-Dim plot.

The surplus is the further utility gained from the utility at value of actual expenditure (at equilibrium price).



In our new theory, there are 3 coordinates. Existing consumer surplus does not fit into our theoretical foundation. We argue that the surplus area should not be interpreted as utility and it should be thought in beyond the 2-Dim plane to the 3rd coordinate φ as the below picture.

Utility (as heat) is not area under curve in the plane of mechanical pair. Utility should be the product of Economic Temperature φ and Economic Entropy S^d . (as T - S diagram)



TOTAL GEN. UTILITY, GEN. SURPLUS

Consider enthalpy

$$H = U - XY = TS$$

$$dH \leq TdS - XdY$$

$$\mathcal{H} = \mathcal{W} + PrQ^d = \varphi S^d$$

$$d\mathcal{H} \leq \varphi dS^d + Q^d dPr$$

At Pr^* , our new theory suggest the enthalpy $\mathcal{H}$ or $\varphi^* S^{d*}$ as consumer's Total Gen. Utility at equilibrium

“Generalized Surplus is hence Total Gen. Utility subtracted by Expenditure”

Generalized Surplus is hence $\mathcal{W}$

At fixed equilibrium price, if there is a change in S^d (utility per personal wealth), e.g. the poorer has higher S^d , increasing in Total Gen. Utility (thermal energy φS^d) is always less than Gen. Utility (heat) production

Inequalities awaiting to be interpreted $d\mathcal{H} \leq \varphi dS^d$

HELMHOLTZ FREE ENERGY

Consider enthalpy

$$\mathcal{F} = U - TS = -PV$$

$$d\mathcal{F} \leq YdX - SdT$$

$$\mathcal{F} = \mathcal{W} - \varphi S^d = -PrQ^d$$

$$d\mathcal{F} \leq -PrdQ^d - S^dd\varphi$$

Hence at Pr^* , Helmholtz Free Energy is the expenditure

•

Inequalities awaiting to be interpreted and proved empirically,
although hard to control in real economy!

FURTHER WORKS

This is the beginning step to

- Applying Legendre transformation to find other potential and find economics meaning
- Using the EoS with other existing literature in the field of statistical mechanics in finance in which partition function and Hamiltonian are without solid ground of demand-side economics mechanical pair (price and demand quantity) **found here with help of effect structure**. This gives contribution to financial physics modelling
- Trying with real data | confronting with traditional economics prediction
- Concepts of I.C. curve and multiple EoS
- Heat engine
- Supply-side

SOME KEY REFERENCES

- I. Fisher. Mathematical investigations in the theory of value and prices. Doctoral Thesis, vol. IX, Trans. of the Connecticut Acad. (1892).
- P. A. Samuelson. Structure of a minimum equilibrium system. In: R. W. Pfouts (Ed.), Essays in Econ. Econometrics: A Volume in Honor of Harold Hotelling. University of North Carolina Press (1960). Reprinted in J. E. Stiglitz, J.E. (Ed.). The Collected Scientific Papers of Paul A. Samuelson. MIT Press, pp. 651 (1966).
- P. Samuelson. Gibbs in economics. In: G. Caldi and G. Mostow (Ed.), Proceedings of Gibbs Symposium. American Mathematical Society (1990).
- G. Debrue. Theory of Value. Yale University Press (1972).
- H. Buchdahl. The Concepts of Classical Thermodynamics, Cambridge University Press (1966).
- B. Gumjudpai. Towards equation of state for a market: A thermodynamical paradigm of economics. Jour. Phys.: Conf. Ser. 1144, 012181 (2018) [arXiv: 1807.09595].
- B. Gumjudpai and Y. Setthapramote. Nature of thermodynamics equation of state towards economics equation of state. Proc. 11th Silpakorn University Research Fair. Silpakorn University, Thailand (2019).
- B. Gumjudpai and Y. Setthapramote. Effect structure in physics and hints to economics equation of state. Jour. Phys.: Conf. Ser. (In Press).
- L. Walras. Economique et mécanique. Bull. Soc. Vaudoise Sci. Nat. 45, 313 (1909)
- D. G. Luenberger. Microeconomic Theory. McGraw-Hill (1995).
- A. Mas-Colell, M. D. Whinston and J. R. Green. Microeconomic Theory. Oxford University Press (1995).
- A. Mas-Colell. The Theory of General Economic Equilibrium: A Differentiable Approach. Cambridge University Press (1985).
- J. H. C. Lisman. Econometrics, Statistics and Thermodynamics. The Netherlands Postal and Telecommunications Services. Holland (1949).
- E. Smith, D. K. Foley. Classical thermodynamics and economic general equilibrium theory, J. Econ. Dyn. Contr. 32, 7 (2008).
- W. M. Saslow. An economic analogy to thermodynamics. Am. J. Phys. 67, 1239 (1999).
- A. Münster. Classical Thermodynamics. Wiley-Interscience (1970).
- R. Giles. Mathematical Foundations of Thermodynamics. Pergamon Press (1964).
- D. N. Gujarati and D. C. Porter. Basic Econometrics 5th Ed. McGraw-Hill (2009).

THANK YOU FOR ATTENTIONS

*Burin
Gumjudpai*