

# Timeline of Econophysics

Bikas K. Chakrabarti

*in collaboration with:*

S. Mukherjee

A. Sinha

Saha Institute of Nuclear Physics



**1931:** M. N. Saha & B. N. Srivastava in their textbook “A Treatise on Heat” [Indian Press, Allahabad (1931), pp. 104-105], in the section on Maxwell-Boltzmann velocity distribution in ideal gases, suggested the use of statistical physics to explain the (assumed) Gamma-like income/wealth distribution in any country. M. N. Saha published several articles on scientific study of social problems in his founded and edited journal Science & Culture [see, e.g., B. K. Chakrabarti, “Econophysics as Conceived by Meghnad Saha”, Science & Culture, Vol. 84 (2018), pp. 365-369] and was the founder of Saha Institute of Nuclear Physics, Kolkata (named after his death).

# Developments Up To 2000 Contd.

$i_A$ , the pressure exerted by the mixture is equal to the sum of the pressures exerted separately by its several components. This is Dalton's law of partial pressures.

**3-15. Introduction of Temperature.**—If we consider a gramme-molecule of the gas which occupies a volume  $V$ , we get from (10),

$$pV = \frac{1}{3}MC^2, \quad (16)$$

$M$  being the molecular weight. In order to introduce temperature in the foregoing kinetic consideration of a perfect gas we have to make use of the experimental law,  $pV = RT$ . Hence

$$RT = \frac{1}{3}MC^2, \quad C^2 = \frac{3RT}{M}. \quad (17)$$

Thus  $C^2$  is proportional to the absolute temperature which may thus be considered proportional to the mean kinetic energy of translation of the molecules. This is the kinetic interpretation of temperature. Hence, according to the kinetic theory, the absolute zero of temperature is the one at which the molecules are devoid of all motion. This deduction is, however, not quite justified since the perfect gas state does not hold down to the absolute zero. The interpretation given by classical thermodynamics is somewhat different (Chap. VI) and more reasonable. That does not necessarily require that all motion should cease at the absolute zero. Further, according to the quantum theory the substance may have some energy at the absolute zero which is called the zero-point energy (Nobelpreize).

We can put  $M=Nm$  where  $m$  is the mass of a single molecule and  $N$  the number of molecules in a gram-molecule, which is usually called the *Avogadro number*. Further let us put  $R/N=k$  where  $k$  is known as *Boltzmann's constant*. Then we have

$$p = \frac{1}{3}(N/V)kT = nkT, \quad (18)$$

where  $n$  is the number of molecules per c.c. and is known as *Loschmidt's number*. Further, from (17)

$$\frac{1}{2}Nm\bar{v}^2 = \frac{1}{2}NkT, \quad \frac{1}{2}m\bar{v}^2 = \frac{1}{2}kT, \quad (19)$$

i.e., the mean kinetic energy of translation of one molecule is  $\frac{1}{2}kT$ .

**3-16. Distribution of Velocities.**—In the previous sections we deduced the gas laws from very general considerations. We assumed that though the particles are incessantly colliding and having their velocities changed constantly in direction as well as in magnitude, we can still talk of a steady state in which  $n_x$ , the number of molecules having the velocity component  $u_x$ , cannot have its value changed<sup>1</sup> by collisions. The law which expresses  $n_x$  in terms of  $u$  and other known quantities is called the *law of distribution of velocities* and was first arrived at by Maxwell in 1859. Before proceeding

<sup>1</sup> This is proved in Note 2 from a detailed consideration of the collision between the molecules.

to deduce the law we shall explain more fully the nature of this law. It will be noted that such an assembly of molecules is a *statistical* system and the problem of finding the distribution of velocities among the molecules is essentially a statistical problem. We shall therefore illustrate the problem by a simple statistical analogy.

Suppose in a country the assessing department is required to find out the average income per head of the population. They will proceed somewhat in the following way. They will find out the number of persons whose income lies within different small ranges. For example, they will find out the number of persons whose income lies between 10s. and 11s., between 11s. and 12s. and so on. Instead of a shilling, they may choose a smaller interval, say 6d. Then it can be easily seen that the number of persons whose income lies between 10s. and 10s.6d. will be approximately half the number found previously for the range 10s. to 11s. We can generalize by saying that the number whose income lies between  $x$  and  $x+dx$  is  $n_x dx$ . It should be noted that the number is proportional to the interval chosen ( $dx$ ). To get the average income they should choose the interval to be as small as possible, say a penny. When this is not possible they will choose a bigger interval but their results will be proportionately inaccurate.

To represent graphically the income of the population they will plot a curve with  $x$  as ordinate and  $n_x$  as abscissa. The curve will be similar to that given in Fig. 6. This will begin with a minimum at 0, rise to a maximum at some point, and thereafter approach the axis of  $x$ , meeting it at a great distance. The curve will have this shape because the number of absolute beggars is very small, and the number of millionaires is also small, while the majority of population have average income.



Fig. 6.—Distribution of income among persons.

Let us now return to the problem of a perfect gas enclosed in a vessel. Let it be devoid of all mass motion or convection currents. Following Boltzmann we make the simplifying assumption that in the equilibrium state the gas is in the state of *molecular chaos*. This assumption states that all possible states of motion occur with equal frequency. This really comprises two assumptions<sup>2</sup>—(1) The number  $n$  of molecules per c.c. remains constant on the average in course of time in every element of volume throughout the

<sup>1</sup> It may be pointed out that for most purposes it is enough to be able to state the number of persons having a certain income and we need not find which person has got what income. Similarly with the molecules. It is quite sufficient to say that so many molecules have so much velocity and so on, which molecule has got what velocity is of little importance and the more so because we do not see the molecules.

<sup>2</sup> For an analysis of the assumptions underlying the hypothesis of molecular chaos see Jeans, *Kinematic Theory of Gases*, (1915) pp. 10-14. The assumption of molecular chaos seems plausible but is nevertheless open to criticism. However it can be shown from statistical mechanics that in the state of equilibrium this assumption does hold during practically the whole of the time.

## Developments Up To 2000 Contd.

- ▶ The number density of particles  $P(\epsilon)$  having energy between  $\epsilon$  to  $\epsilon + d\epsilon$  is given by  $P(\epsilon)d\epsilon = g(\epsilon)f(\epsilon)d\epsilon$ . In three dimension momentum counting gives the density of states  $g(\epsilon) \sim \sqrt{\epsilon}$ . Energy conservation during the scattering process suggests  $f(\epsilon_1)f(\epsilon_2) = f(\epsilon_1 + \epsilon_2)$  for arbitrary values of  $\epsilon_1, \epsilon_2$ . This leads to the energy distribution  $f(\epsilon) \sim \exp(-\epsilon/\Delta)$ ;  $\Delta$  is a constant. Together these lead to  $P(\epsilon) \sim \sqrt{\epsilon} \exp(-\epsilon/\Delta)$ .
- ▶ In a market where each two-agent (two-body) trade/scattering keeps total money conserved the money distribution should also satisfy  $f(m_1)f(m_2) = f(m_1 + m_2)$ , leading to  $f(m) \sim \exp(-m/\delta)$ , where  $\delta$  is constant. In absence of any momentum equivalent, the  $g(m)$  is constant here, leading  $P(m)$  to be an exponentially decaying function.
- ▶ Saha and Srivastava (1935) asked students to think how this exponential  $P(m)$  becomes more like Gamma distribution.

## Developments Up To 2000 Contd.

**1942:** E. Majorana wrote a detailed Note emphasizing on “The Value of Statistical Laws in Physics & in Social Science”, published in 1942 after he went missing in 1938 [translated in to English in 2005, from Italian, by R. N. Mantegna, published in Quantitative Finance, Vol. 5, p. 135-140 (2005); see also R. N. Mantegna, in Proceedings of the XXXIII Congress of the Italian Society for the History of Physics and Astronomy (SISFA 2013); <https://arxiv.org/pdf/1409.0789> and T. Gianfranco, “From Galileo to Modern Economics: The Italian Origins of Econophysics”, Palgrave Macmillan, (2018)].

**1960:** B. Mandelbrot published “The Pareto-Levy Law & the Distribution of Income”, International Economic Review, Vol. 1 (1960), pp. 79-106, arguing for statistical mechanical modelling of markets.

## Developments Up To 2000 Contd.

**1963:** B. Mandelbrot published “The Variation of Certain Speculative Prices”, Journal of Business, Vol. 36 (1963), pp. 394-419, indicating the failure of the Random walk statistics in capturing the Stock price fluctuations.

**1971:** L. P. Kadanoff published the paper “From Simulation Model to Public Policy: An Examination of Forrester’s “Urban Dynamics””, Transactions of the Society for Computer Simulation, Vol. 16 (1971), pp. 261-268, arguing for statistical physical analysis of social dynamics.

**1974:** E. W. Montroll & W. W. Badger published the book titled “Introduction to Quantitative Aspects of Social Phenomena”, Gordon and Breach, Washington (1974). Later, E. W. Montroll also published “On the Dynamics & Evolution of Some Sociotechnical Systems”, Bulletin of the American Mathematical Society, Vol. 16 (1987), pp. 1-46, again emphasizing the importance of statistical physics in modelling of social dynamics.

## Developments Up To 2000 Contd.

**1995:** B. K. Chakrabarti & S. Marjit published “Self- Organization & Complexity in Simple Model Systems: Game of Life & Economics”, Indian Journal of Physics B, Vol. 69 (1995), pp. 681-698, discussing in particular the classical (and quantum-like) statistical mechanics of market models and the consequent nature of income or wealth distributions [this paper being the first Indian one with both physicist and economist coauthors]. H. E. Stanley coined the term ‘Econophysics’ in the Statphys-Kolkata II Conference 1995 [Series initiated by the Saha Institute of Nuclear Physics & S. N. Bose National Centre for Basic Sciences; X-th Conference held in 2019] and published in its Proceedings [H. E. Stanley et al., “Anomalous Fluctuations in the Dynamics of Complex Systems: From DNA & Physiology to Econophysics”, Physica A: Statistical Mechanics and its Applications, Vol. 224 (1996), pp. 302-321].



ELSEVIER

Physica A 224 (1996) 302-321

PHYSICA A

## Anomalous fluctuations in the dynamics of complex systems: from DNA and physiology to econophysics

H.E. Stanley<sup>a</sup>, V. Afanasyev<sup>b</sup>, L.A.N. Amaral<sup>a</sup>, S.V. Buldyrev<sup>a</sup>,  
A.L. Goldberger<sup>c,d</sup>, S. Havlin<sup>a,e</sup>, H. Leschhorn<sup>a</sup>, P. Maass<sup>a</sup>,  
R.N. Mantegna<sup>f</sup>, C.-K. Peng<sup>a,c</sup>, P.A. Prince<sup>b</sup>, M.A. Salinger<sup>g</sup>,  
M.H.R. Stanley<sup>a</sup>, G.M. Viswanathan<sup>a</sup>

<sup>a</sup> Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA

<sup>b</sup> British Antarctic Survey, Natural Environment Research Council, High Cross, Madingley Road, CB3 0ET, Cambridge, UK

<sup>c</sup> Cardiovascular Division, Harvard Medical School, Beth Israel Hospital, Boston, MA 02215, USA

<sup>d</sup> Department of Biomedical Engineering, Boston University, Boston, MA 02215, USA

<sup>e</sup> Minerva Center and Dept. of Physics, Bar-Ilan University, Ramat Gan, Israel

<sup>f</sup> Dipartimento di Energetica ed Applicazioni di Fisica, Palermo U., Palermo, I-90128, Italy

<sup>g</sup> School of Management, Boston University, Boston, MA 02215, USA

### Abstract

We discuss examples of complex systems composed of many interacting subsystems. We focus on those systems displaying nontrivial long-range correlations. These include the one-dimensional sequence of base pairs in DNA, the sequence of flight times of the large seabird *Wandering Albatross*, and the annual fluctuations in the growth rate of business firms. We review formal analogies in the models that describe the observed long-range correlations, and conclude by discussing the possibility that behavior of large numbers of humans (as measured, e.g., by economic indices) might conform to analogs of the scaling laws that have proved useful in describing systems composed of large numbers of inanimate objects.

## Developments Up To 2000 Contd.

**1999:** R. Mantegna & H. E. Stanley published “Scaling Behaviour in the Dynamics of an Economic Index”, *Nature*, Vol. 376 (1999), pp. 46-49. Also, R. Mantegna & H. E. Stanley “An Introduction to Econophysics”, Cambridge University Press, Cambridge (1999).

**2000:** A. Dragulescu & V. M. Yakovenko published “Statistical Mechanics of Money”, *European Physical Journal B*, Vol. 17 (2000), pp. 723-729; A. Chakraborti and B. K. Chakrabarti published “Statistical Mechanics of Money: How Saving Propensity Affects its Distribution” *European Physics Journal B*, Vol. 17 (2000), pp. 167-170; and J.-P. Bouchaud and M. Mzard published “Wealth Condensation in a Simple Model of Economy”, *Physica A: Statistical Mechanics and its Applications*, Vol. 282 (2000), pp. 536-545. All these three papers employed ideal gas-like model in which each agent is represented by a gas molecule in a trading (modeled as collision) with money (modeled as energy) to investigate the emerging features of income & wealth distributions.

# Developments Up To 2000 Contd.

## Market Exchange Scattering Process

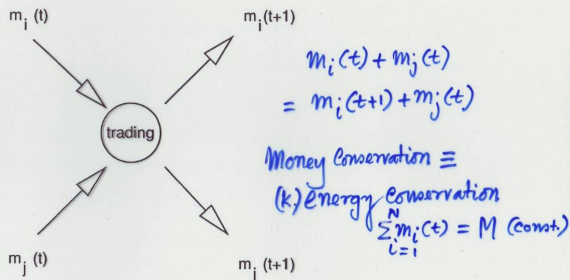


FIG. 2: Schematic diagram of the trading process. Agents  $i$  and  $j$  redistribute their money in the market:  $m_i(t)$  and  $m_j(t)$ , their respective money before trading, changes over to  $m_i(t+1)$  and  $m_j(t+1)$  after trading.

$$m_i(t+1) = \lambda_i m_i(t) + \epsilon [(1-\lambda_i) m_i(t) + (1-\lambda_j) m_j(t)]$$
$$m_j(t+1) = \lambda_j m_j(t) + (1-\epsilon) [(1-\lambda_i) m_i(t) + (1-\lambda_j) m_j(t)]$$

random fraction  
(annealed)

saving propensity  
of  $i$ -th agent  
(quenched)

# Developments Up To 2000 Contd.

## Steady State Money Distribution $P(m)$

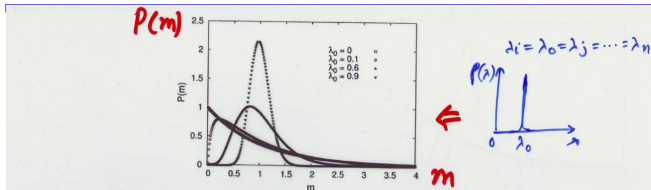


FIG. 3: Steady state money distribution  $P(m)$  for the model with uniform savings. The data shown are for different values of  $\lambda$ : 0, 0.1, 0.6, 0.9 for a system size  $N = 100$ . All data sets shown are for average money per agent  $M/N = 1$ .

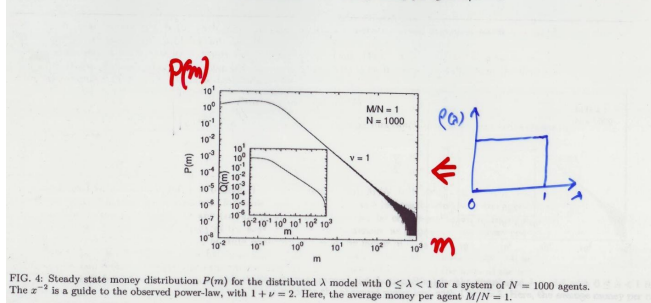


FIG. 4: Steady state money distribution  $P(m)$  for the distributed  $\lambda$  model with  $0 \leq \lambda < 1$  for a system of  $N = 1000$  agents. The  $x^{-2}$  is a guide to the observed power-law, with  $1 + \nu = 2$ . Here, the average money per agent  $M/N = 1$ .

## Reviews

2007: A. Chatterjee & B. K. Chakrabarti (2007), “Colloquium: Kinetic Exchange Models for Income & Wealth Distributions”, European Physical Journal B, Vol. 60, pp. 135-149.

2009: V. M. Yakovenko & J. Barkley Rosser Jr. (2009), “Colloquium: Statistical Mechanics of Money, Wealth, and Income”, Reviews of Modern Physics, Vol. 81, pp. 1703-172.

2011: A. Chakraborti, I. Muni Toke, M. Patriarca & F. Abergel (2011), “Econophysics Review I: Empirical Facts”, Quantitative Finance, Vol. 11, pp. 991-1012; “Econophysics Review II: Agent-based Models”, Quantitative Finance, Vol. 11, pp. 1013-1041.

# Developments beyond 2000 Contd.

## Books

2005: D. Challet, M. Marsili & Y.-C. Zhang (2005), *Minority Games*, Oxford University Press, Oxford.

2010: S. Sinha, A. Chatterjee, A. Chakraborti & B. K. Chakrabarti (2010) *Econophysics: An Introduction*, Wiley-VCH, Weinheim.

2013: B. K. Chakrabarti, A. Chakraborti, S. R. Chakravarty & A. Chatterjee (2013), *Econophysics of Income & Wealth Distributions*, Cambridge University Press, Cambridge.

2013: F. Slanina (2013), *Essentials of Econophysics Modelling*, Oxford University Press, Oxford.

2013: P. Richmond, J. Mimkes & S. Hutzler (2013), *Econophysics & Physical Economics*, Oxford University Press, Oxford.

2014: Pareschi & Toscani (2014), *Interacting Multiagent Systems*, Oxford Univ. Press, Oxford.

# Developments beyond 2000 Contd.

2016: M. Shubik & E. Smith (2016), The Guidance of an Enterprise Economy, MIT Press, Massachusetts.

2017: H. Aoyama, Y. Fujiwara, Y. Ikeda, H. Iyetomi, W. Souma & H. Yoshikawa (2017), Macro-Econophysics, Cambridge University Press, Cambridge.

2017: F. Jovanovic & C. Schinckus (2017), Econophysics and Financial Economics, Oxford Univ. Press, New York.

2017: B. K. Chakrabarti, A. Chatterjee, A. Ghosh, S. Mukherjee, & B. Tamir (2017), Econophysics of the Kolkata Restaurant Problem and Related Games: Classical and Quantum Strategies for Multi-agent, Multi-choice Repetitive Games, New Economic Windows, Springer, Cham.

# Developments beyond 2000 Contd.

## **Thesis**

2018: C. Schinckus (2018), Thesis entitled “When Physics Became Undisciplined: An Essay on Econophysics”, Department of History and Philosophy of Science, University of Cambridge (Freely available online).

## **Popular Reports and Writings**

2002: B. Hayes (2002), “Follow the Money”, American Scientist, Vol. 90, pp. 400-405.

2005: J. Hogan (2005), “Why it is hard to share the wealth?”, New Scientist, 12 March 2005, p. 6.

2007: A. Chatterjee, S. Sinha & B. K. Chakrabarti (2007), “Economic inequality: Is it natural?”, Current Science, Vol. 92 (2007) pp. 1383-1389.

# Developments beyond 2000 Contd.

2007: M. Buchanan (2007), “The Socail Atom”, Bloomsbury, New York.

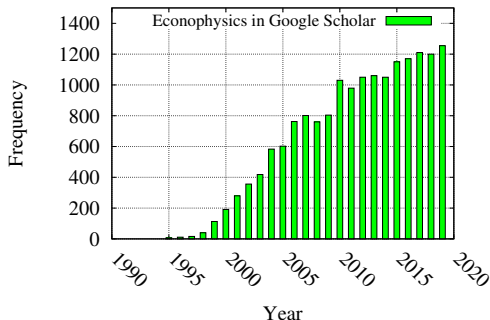
2012: S. Battersby (2012), “Physics of our Finances”, New Scientist, 28 July 2012, p. 41.

2012: S. Sinha & B. K. Chakrabarti (2012), “Econophysics: An Emerging Discipline”, Economic & Political Weekly, Vol. 46 (2012), pp. 44-65.

2019: K. C. Dash (2019), “The Story of Econophysics”, Cambridge Scholars Publishing, Newcastle upon Tyne.

2019: B. M. Boghosian (2019), “Is Inequality Inevitable?”, Scientific American, Vol. 321, pp. 70-77.

# Econophysics Citation Histogram



Histogram plot of numbers of entries per year containing the term econophysics versus the corresponding year. The data are taken from Google Scholar (dated: 31 December, 2019). It may also be noted from Google Scholar that while this 24-year old econophysics has today typical yearly citation frequency of order  $1.2 \times 10^3$ , more than 100-year old subjects like astrophysics (Meghnad Saha published his ionization equation for solar chromosphere in 1920), biophysics (Karl Pearson coined the term in his 1892 book 'Grammar of Science') and geophysics (Issac Newton explained planetary motion, origin of tides, etc in 'Principia Mathematica', 1687) today have typical yearly citation frequencies of order  $36.6 \times 10^3$ ,  $40.2 \times 10^3$  and  $45.4 \times 10^3$  respectively.

# Thank You