

## DIFFERENTIAL FORMS IN THERMODYNAMICS AND ECONOMICS

Juergen Mimkes

Physics Department, Paderborn University, Paderborn, Germany

### ABSTRACT

Alternating differential forms introduced by Élie Cartan (1894) have been successfully used in science. In this contribution differential forms in two dimensions are applied to thermodynamics depending on pressure and volume; and to economics depending on capital and labor.

1. In two-dimensional calculus we have complete differential forms ( $dE$ ) and incomplete differential forms ( $\delta Q$ ). Complete differentials may be integrated by a Riemann integral, which lead to the stem function  $\epsilon$ . Incomplete differentials do not have a stem function, the Stokes integral is path dependent.

2. In thermodynamics a conservative term like energy may be represented by a complete differential form ( $dE$ ). A not-conservative term like heat may be represented by an incomplete differential form ( $\delta Q$ ). The observation “*Heat is created by frictional work*”,  $\oint \delta Q = - \oint \delta W$ , leads to the first and second laws of thermodynamics.

3. In economics, complete differential forms ( $dK$ ) correspond to predictable (ex-ante) terms like interest from a savings contract. Incomplete differential forms ( $\delta Y$ ) represent not-predictable (ex-post) terms like income from stock markets or invested labor - ( $\delta L$ ). The observation “*Income is created by labor*”,  $\oint \delta Y = - \oint \delta L$ , leads to the differential laws of economics.

4. It may be surprising to find the differential laws of thermodynamics the same as the differential laws of economics. But this must be true, as it is now common to apply statistical mechanics to problems of economics. Thermodynamics is closely connected to most other natural sciences. In the same way economics is closely related to most other social sciences. Accordingly, this correspondence applies more general to all natural and social sciences. This may be confirmed by applying the laws in social problems. Natural sciences and social sciences follow the same equations, only the objects and the interactions are different: Atoms follow electro-magnetic fields, people follow emotions. Both types of interactions can be attractive, indifferent or repulsive.

Keywords: differential forms, income, heat, energy, capital, work, labor, electro-magnetic fields, emotions