

AXIOMATIC CHARACTERIZATION OF STATISTICAL COMPLEXITY MEASURES

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ABSTRACT

In the past, different definitions of statistical complexity were introduced based on different definitions of generalized entropies and generalized certainty measures, as an interplay between order and disorder levels of the system. However, there is no general agreement on which one is preferable since it has still not been accepted what constitutes a set of basic properties that a complexity measure should satisfy. In this work, we provide a general treatment of the statistical complexity measures starting from a set of ineluctable properties, which are stated as axioms, by which the statistical complexity measure of a discrete system is defined as a non-negative function of entropy and certainty measures, which is decomposable and attains the minimal value, in the case of fully ordered and fully disordered systems, which are considered to be the simplest one. Moreover, we require that the measure has a finite and non-vanishing limit when a discrete system tends to a continuous one, being invariant under rescaling, translation, and replication. Thus, we propose a generalized statistical complexity measure for discrete systems and differential generalized statistical complexity for continuous systems, which include some of the previously considered measures as special cases and which satisfy all the axioms. Some open questions for future research are pointed out.

Keywords: statistical complexity measures, generalized entropies, certainty measures, strongly pseudo-additive entropies, order and disorder