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THERMODYNAMICS FORMULATION OF ECONOMICS

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ABSTRACT

We consider demand-side economy. Using Caratheodory approach, we define empirical existence of equation of state (EoS) and coordinates. We found new insights of thermodynamics EoS, "the effect structure". Rules are proposed as criteria in promoting and classifying an empirical law to EoS status. Four laws of thermodynamics are given for economics. We proposed a method to model the EoS with econometrics. Consumer surplus in economics can not be considered as utility. Concepts such as total wealth, generalized utility and generalized surplus are introduced. EoS provides solid foundation in statistical mechanics modelling of economics and finance.

INTRODUCTION

There exist equilibrium states in economics and thermodynamics both of which describe aggregated phenomena. We criticize in how thermodynamical states are in existing. Key concepts are of the thermodynamics coordinate space and the equation of state (EoS). Thermodynamics space is $2(n+1)$ tuples of (X_i,Y_i,S,T) where X_i,Y_i are extensive and intensive coordinates and $i=1\ldots n$. At equilibrium S is fixed, the manifold becomes $(2n+1)$ tuples, $\mathscr{M}=(X_i,Y_i,T)$. Existing of the manifold space is supported by empirical axioms , the existence of thermal states and entropic states (Münster 1970 [1], Landé 1926 [2]). These states relate two mechanical pair variables giving rise to the EoS, $g(X_i,Y_i,T)=0$. For $n=1$ in hydrostatics system, this is $g(V,P,T)=0$ space. The EoS constrained the system to evolve on a 2-Dim surface. On the surface, thermodynamics potential as analogy to a two degree of freedom "field". For example, the internal energy, $U=U(X,Y)=U(Y,T)=U(X,T)$. With Legendre transformation, it can be transformed to other potentials. Energy transfer in thermodynamics comes in form of work, $\delta W = Y\mathrm{d}X$ and heat, $\delta\mathscr{Q}=T\mathrm{d}S$. In economics, there are stock quantity or flow quantity. We should classify which stock variables should be potential or should be coordinates. The flow is analogous to energy transfer. There is no concept of EoS and potential in economics (Debrue 1972 [3], Mas-Colell 1985 [4]). However similarities in common are: (1) equilibrium determined by a set of pari of dual state variables, (2) constraints of conservation, (3) preference of a quantity not to decrease in approaching equilibrium. Following these similarity, Smith and Foley 2008 [5] proposed $g(Q^{\mathrm{d}},Pr,\mathrm{MRS})=0$ as EoS of consumers where $Q^{\mathrm{d}},Pr,\mathrm{MRS}$ are demand quantity, price and marginal rate of substitution (among more than one type of commodity). Previous attempts has long history back to Fisher's thesis (Fisher 1892 [6]) which considered gradient of utility with respect to quantity of goods as analogy to gradient of internal energy. Later, mechanical gradient of potential energy analogy to gradient of utility with respect to quantity of goods was considered by Walras (Walras (1909) [7]). This had been abandoned for many years until Samuelson's critique (Samuelson 1960 [8]). There are more investigations by Lisman (Lisman 1949 [9]) and Saslow (Saslow 1999 [10]) in connecting thermodynamics variables to economics however the EoS was not considered therein. A few aspects in comparison are noticed as follow. In physics: (1) manifold as home of EoS and manifold coordinates are (X,Y,T) (2) prediction

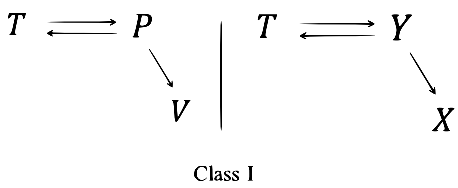

FIGURE 1. EFFECT STRUCTURE DIAGRAM OF A CLASS I EOS: EXAMPLE IS (V,P,T) SYSTEM.

power, (3) looking for unification, (4) short relaxation time and controlled experiment, (5) potential (or field), (6) more fundamental units (meter, second, kg, Coulomb, Kelvin, mole). In economics: (1) no EoS and manifold coordinates are quantities of each type of goods (Q_j^{d}) (2) empirical formulae with prediction power, (3) unification is as crucial but separated by schools of thoughts, (4) long relaxation time - hardly attain equilibrium state (frequently with minor disturbance or big shocks), (5) no concept of potential (or field), (6) fewer fundamental units (money unit, time unit, quantity of goods, number of agents (consumer, seller, company, etc.))

Systems or agents of both subjects have (1) name, (2) properties (elasticity), (3) duty according to its properties (governed by laws), (4) systematic uncertainty (systematic risk). However only in social system or agents that possess ability to make choice or option which is based on many factors such as memory, preference etc.

Here we propose new idea to thermodynamics, the effect structure of an EoS and apply this to economics, such that econometric modeling is to be guided by EoS effect structure idea. We hope to propose a concise and more complete picture of "thermodynamical paradigm" of economics.

TRULY ENDOGENOUS FUNCTION AND EFFECT STRUCTURE DIAGRAM

In this section, we develop concept of the truly endogenous function and effect structure diagram as machinery tools for inductive reasoning toward generalized rules or statement for an empirical equation to attain a status of EoS. This can be done by noticing common nature of relevant laws and theories. In simple EoS such as ideal gas law, there are directions of effect as concluded. (a) firstly, pressure P can affect temperature T or volume V directly whereas initial change in P comes from externality. (b) secondly, T can affect P directly whereas initial change in T is from externality. More considerations are that T can not affect V directly, but only via P and that V can affect neither T nor P at all. Change in V can not be done by other external variables but can only be done by P, hence V is always passive. Effects from externality is dubbed exogenous. Only T and P can take the exogenous effect. Functions related internal EoS variables (P,V,T) is endogenous functions. Only the functions represent the effects in (a) and (b) are truly endogenous functions. When writing $V=V(P,T)$, it looks fine. However in this consideration, it is in fact $V=V\circ P(T)=V(P(T))$. Hence T is exogenous for V. Hence for a hydrostatics system truly endogenous functions (denoted with tilde sign) are:

$$V=\tilde{V}(P), \qquad P=\tilde{P}(V), \qquad T=\tilde{T}(P). \tag{1}$$

Argument of function is the cause and the value of function is the effect.

Class I and Class II Diagrams

These truly endogenous functions can be represented as directed graphs in Fig. 1. Arrows represent truly endogenous function as they represent direction of cause and effect. Initial change in T or in Y are from exogenous effect, i.e. not from the variables within the diagram. We shall call the diagram with two opposite arrows linking T and Y, the Class I diagram of EoS. Considering a paramagnetic substance[1], the EoS is $M=CB_0/T$ where M,C,B_0 are magnetization (extensive), Curie constant and external magnetic field intensity (intensive). T and M can affect each others however B_0 can not be affected by neither M nor T as shown in Fig. 2. The diagram is with two opposite arrows linking T and X hence it shall be realized as Class II diagram of EoS.

A Proposal of Effect Structure Diagram Rules for EoS Status

After observation of many EoS in physical nature, we inductively form a set of rules as criteria for judging status of an EoS, $g(X,Y,T)=0$. These are

1. Number of the arrows is three.
2. There is at least one arrow pointing $Y \to X$.
3. Only T and(or) Y (apart from causing truly endogenous effects) can also taken exogenous influence or a shock. X can not take exogenous effect unless via Y or T[2].

DEMAND-SIDE ECONOMY

Assuming a one commodity market in perfect competitive condition with information symmetric and market

[1] the case of paraelectric material is similar. Intensive quantity is electic field intensity and expensive quantity is electrical polarization.

[2] X (e.g. volume) can be independent variable in the equation that related Y and T, i.e. $Y=Y(T(X))=Y\circ T(X)$ or $T=T(Y(X))=T\circ Y(X)$ so that changing X might look like exogenous effect. However for a system in reality, given a value of X at beginning, one can not change value of X unless with truly endogenous effect from Y and T.

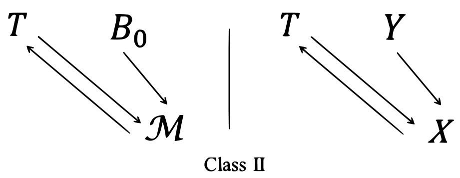

FIGURE 2. EFFECT STRUCTURE DIAGRAM OF A CLASS II EOS: EXAMPLE IS (M,B,T) SYSTEM.

clearance at equilibrium, we propose demand quantity Q^{d} as X coordinate, price Pr as Y coordinate and average personal wealth φ as T. In reality, Q^{d} takes passive role and is extensive with unit of number (amount) of goods. Typically, intensive coordinate Y has a nature of force or influence per unit area or unit entity such as unit of charge. Here Pr is the cost of good per unit good, hence has money unit per unit of good. For ideal system, T is in fact proportional to internal energy per mole or per particle, e.g. $T = 2U/(3Nk_{\mathrm{B}})$ in ideal gas. Hence T is related to the variable defined by the first law, that is the internal energy, U. The unit of energy is analogous to unit of money in economics. Therefore total wealth function $\mathscr{W}$ (in money unit) should play a role as U. As a consequence, average personal wealth φ which plays a role of temperature should be defined as $\varphi = \mathscr{W}/N$ where N is number of consumers in the system. The total wealth function $\mathscr{W}$ comes in two parts. It is a combination,

$$\mathscr{W} \;=\; \text{wealth in happiness} \;+\; \text{wealth in asset}. \tag{2}$$

As this setting, $\mathscr{W}$ is a field potential, $\mathscr{W} = \mathscr{W}(Q^{\mathrm{d}}, Pr, \varphi)$. To strictly define space $(Q^{\mathrm{d}}, Pr, \varphi)$, one needs to consider Carathéodory's axioms which consider empirical existence of a set of infinite "thermal" states, $f(Pr_1, Q^{\mathrm{d}}_1) = \varphi_1$ and $f(Pr_2, Q^{\mathrm{d}}_2) = \varphi_2$ and so forth. and existence of a set of infinite "entropic" states, $s(Pr_1, Q^{\mathrm{d}}_1) = S^{\mathrm{d}}_1$ and $s(Pr_2, Q^{\mathrm{d}}_2) = S^{\mathrm{d}}_2$ and so forth. This allows EoS $g(Q^{\mathrm{d}}, Pr, \varphi) = 0$ to exist. The EoS reduces one degree of freedom of $\mathscr{W}$. Hence total wealth function is defined only on the EoS 2-Dim. surface.

Mechanical changes in total wealth of consumers is defined by work term (demand-side) of mechanical pair" $(-Pr, Q^{\mathrm{d}})$. The work term $\delta W^{\mathrm{d}} = -Pr\,\mathrm{d}Q^{\mathrm{d}}$ is the expenditure for the generalized utility $\delta\mathscr{Q}^{\mathrm{d}}_{\mathrm{util}}$ which is a heat term, $\delta\mathscr{Q}^{\mathrm{d}}_{\mathrm{util}} = \varphi\,\mathrm{d}S^{\mathrm{d}}$ (reversible process).

Here, unlike microeconomics, maximizing of generalized utility $\delta\mathscr{Q}^{\mathrm{d}}_{\mathrm{util}}$ is not derived by optimizing with respect to quantity of different types of goods, Q^{d}_j (economics space coordinates) under a budget constraint. Instead generalized utility can be maximized with the second law to be stated later. This is achieved even for one commodity consumption from expenditure or it can be achieved even without expenditure δW^{d}, but only with $\mathrm{d}S^{\mathrm{d}} \neq 0$. Concept of generalized utility (heat term) is the combination,

$$\begin{aligned}\delta\mathscr{Q}^{\mathrm{d}}_{\mathrm{util}} \;=\;& \delta(\text{pleasure or opportunity of ownership in assets}) \\ &+\delta(\text{happiness of utilizing commodity})\end{aligned} \tag{3}$$

and the change in S^{d} is hence $\Delta S^{\mathrm{d}} = \int \delta\mathscr{Q}^{\mathrm{d}}_{\mathrm{util}}/\varphi$, i.e. change in generalized utility per unit of average personal wealth.

Class III Diagram

Following effect structure diagram rules, and with observations of fact in economy, the effect structure diagram (Fig. 3) for demand-side economy is of a new class with two opposite-direction arrows linking Y and X, the Class III.1[3].

Zeroth Law

The zeroth law gives the existence of average personal wealth φ. Although hard to measure in society, however it helps defining personal wealth equilibrium when there is thermal (wealth) contact. Two consumers with different personal wealth can share via marriage, partnership, adoption, becoming family or as one household. Equal personal wealth should be approach but slowly. Generalised utility is transferred in the contact (all in money unit). Assets ownership (a sector of generalized utility) can be transferred with or without expenditure, i.e. no work term.

The First Law

The first law gives existence of total wealth function $\mathscr{W} = \mathscr{W}(Q^{\mathrm{d}}, Pr, \varphi)$ with conservation, $\mathrm{d}\mathscr{W} \;=\; \delta\mathscr{Q}^{\mathrm{d}}_{\mathrm{util}} + \delta W^{\mathrm{d}}$

The Second Law

The second law provide existence of entropic function S^{d} interpreted as generalized utility per unit of personal wealth with, $\Delta S^{\mathrm{d}} \geq \int \left(\delta\mathscr{Q}^{\mathrm{d}}_{\mathrm{util}}/\varphi\right)$. In any processes of demand side, utility/(unit of personal wealth) does not decrease. Meaning of S^{d} as utility/(unit of personal wealth) can be understood from the example. Rich people (high personal wealth) have less happiness in using a product. It is cheap for them to buy the product.

The Third Law

At zero personal wealth, utility per unit of personal wealth is zero. It takes infinite steps to reach zero personal wealth. Wealth includes happiness, there is no way to take it away completely. If zero personal wealth exists, i.e. no happiness in any form, there is zero utility for the consumer.

[3] The rules also allow Class III.2-4 of which their effect are as follow: $(X \to T)$, $(T \to Y)$ and $(Y \to T)$.

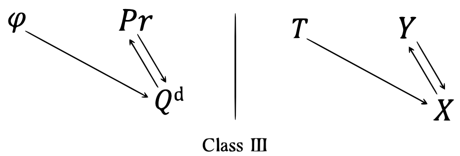

FIGURE 3. EFFECT STRUCTURE DIAGRAM OF A CLASS III.1: DEMAND-SIDE ECONOMY (Q^{d}, Pr, T).

Processes of Demand-Side Economy

Adiabatic process (zero heat), $\mathrm{d}\mathscr{W} = \delta W^{\mathrm{d}} = -Pr\,\mathrm{d}Q^{\mathrm{d}}$ means that there is a spending without gaining any utility. For an economics version of isovolumic process (zero work), $\mathrm{d}\mathscr{W} = \delta \mathscr{Q}^{\mathrm{d}}_{\mathrm{util}} = \varphi\,\mathrm{d}S^{\mathrm{d}}$ Idealistically this is a gaining of utility without spending. Considering isothermal process, there is group of consumers with constant average personal wealth. Processes are governed by Smith's price mechanism, i.e. law of demand. Hence microeconomics is a specific case of our theory. For the last case of isobaric process, i.e. constant price, demand quantity depends on only average personal wealth. If averagely consumers are richer from exogenous effect, demand should increase.

Generalized Consumer Surplus, Total Generalized Utility and Total Expenditure

Consumer surplus does not fit in our foundation. The surplus should not be interpreted as utility and it should be thought in beyond 2-Dim (Pr, Q^{d}) plane to φ axis. Utility (heat) is not area under curve in mechanical pair plane, instead utility should be the product of φ and S^{d}. We suggested that at equilibrium, the total generalized utility φS^{d} is enthalpy, $\mathscr{H} = \mathscr{W} + Pr\,Q^{\mathrm{d}} = \varphi S^{\mathrm{d}}$. Generalized consumer surplus is defined as $\mathscr{H} - Pr\,Q^{\mathrm{d}}$ hence the total wealth $\mathscr{W}$ is the generalized consumer surplus. Increasing in total generalized utility is always less than generalized utility production, i.e. $\mathrm{d}\mathscr{H} \leq \varphi \mathrm{d}S^{\mathrm{d}} + Q^{\mathrm{d}}\mathrm{d}Pr$ for a fixed price. Total expenditure $-Pr\,Q^{\mathrm{d}}$ at equilibrium is the Helmholtz free energy, $\mathscr{F} = \mathscr{W} - \varphi S^{\mathrm{d}}$ (to $F = U - TS$) and $\mathrm{d}\mathscr{F} \leq -Pr\,\mathrm{d}Q^{\mathrm{d}} - S^{\mathrm{d}}\mathrm{d}\varphi$. The inequalities are to be interpreted.

MODELLING DEMAND-SIDE EOS

EoS in general form is $g(Q^{\mathrm{d}}, Pr, \varphi) = 0$. With total differential method, we shall express,

$$\mathrm{d}Q^{\mathrm{d}} = \left(\frac{\partial Q^{\mathrm{d}}}{\partial \varphi}\right)_{Pr} \mathrm{d}\varphi + \left(\frac{\partial Q^{\mathrm{d}}}{\partial Pr}.\right)_{\varphi} \mathrm{d}Pr. \tag{4}$$

$$\beta_{Pr} \equiv \frac{1}{Q^{\mathrm{d}}_0}\left(\frac{\partial Q^{\mathrm{d}}}{\partial \varphi}\right)_{Pr} = \frac{E^{\mathrm{d}}_{\varphi}}{\varphi_0}, \tag{5}$$

$$\kappa_{\varphi} \equiv -\frac{1}{Q^{\mathrm{d}}_0}\left(\frac{\partial Q^{\mathrm{d}}}{\partial Pr}\right)_{\varphi} = -\frac{E^{\mathrm{d}}_{Pr}}{Pr_0}, \tag{6}$$

where E^{d}_{φ} and E^{d}_{Pr} are elasticities of demand to personal wealth and demand to price. In simplest case, elasticities may be assumed constant[4]. It is straightforward to write,

$$Q^{\mathrm{d}}(Pr, \varphi) = Q^{\mathrm{d}}_0\left[1 + \beta_{Pr}(\varphi - \varphi_0) - \kappa_{\varphi}(Pr - Pr_0)\right], \tag{7}$$

which is the EoS for the demand-side system. Further we define $Y \equiv Q^{\mathrm{d}}/Q^{\mathrm{d}}_0$, $X_1 \equiv \varphi - \varphi_0$, $X_2 \equiv Pr - Pr_0$. and our econometric model for the EoS is hence, $Y = 1 + \beta_{Pr}X_1 - \kappa_{\varphi}X_2 + u$, where u is an error term and this can be done with regression analysis in econometrics.

CONCLUSION

We analyse nature of EoS and found effect structure diagrams and its rules for the EoS. We use these criteria to variables in demand-side economy with hopes to express thermodynamics formulation of economics and explore the analogies and interpretation in various aspects.

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