The Role of Thermodynamics in the Control of Networked Systems

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THE ROLE OF THERMODYNAMICS IN THE CONTROL OF NETWORKED SYSTEMS

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ABSTRACT

Network systems involve distributed decision-making for coordination of networks of dynamic agents and address a broad area of applications in science and engineering. In this paper, we develop a thermodynamically-based framework for addressing consensus problems using a hybrid control protocol architecture with a dynamic communication topology wherein communication events are triggered via state-dependent resettings consistent with thermodynamic principles. The proposed hybrid controller architecture involves the exchange of intermittent state information between agents guaranteeing that the closed-loop dynamical network is semistable to an equipartitioned equilibrium representing a state of information consensus consistent with basic thermodynamic principles.

1 INTRODUCTION

The consensus control problem involves information exchange between multiagent networked systems guaranteeing agreement between agents for achieving a given coordination task. Given that this problem addresses a broad area of applications that includes cooperative control of unmanned air vehicles, microsatellite clusters, mobile robotics, battle space management, and congestion control in communication networks, it is not surprising that control of networks and control over networks has attracted considerable attention in the literature [1–3]. Consensus control protocols over static and dynamic information exchange topologies involve neighbor-to-neighbor interactions, wherein agents update their information state based on the information states of the neighboring agents, and have predominately relied on algebraic graph theory [2, 3].

In [4–6], the authors present an alternative new and novel perspective to the consensus control problem that is based on *dynamical thermodynamics* [7, 8]; a framework that unifies the foundational disciplines of thermodynamics and dynamical systems theory. Dynamical thermodynamics was developed in [7, 8] to provide a rigorous foundation for equilibrium and nonequilibrium thermodynamics using a dynamical systems formalism. Dynamical thermodynamics has also been used to apply thermodynamic principles to the analysis and control design of dynamical systems using an energy- and entropy-based hybrid stabilization framework [9–11].

By generalizing the notions of temperature, energy, and entropy, dynamical thermodynamics is used in [4–6] to develop a design procedure for distributed consensus controllers that engender networked dynamical systems to emulate thermodynamic behavior. In particular, for network systems with an undirected and directed communication graph topology, system thermodynamic notions are used to show that every control law protocol of a symmetric Fourier type, with information transfer playing the role of energy flow, achieves information (or communication) consensus [4–6].

In this paper, we develop a hybrid control framework for semistability and consensus of multiagent systems with intermittent information. Specifically, we use impulsive differential equations [9] to construct a hybrid control architecture for addressing network information consensus wherein communication events are triggered via state-dependent resettings. The proposed controller architecture is predicated on the recently developed notion of hybrid dynamical thermodynamics [8, 12] resulting in a hybrid controller architecture involving the exchange of generalized energy state information between agents that guarantee that the closed-loop dynamical network is semistable and consistent with basic thermodynamic principles. Specifically, the hybrid control protocol architecture involves a dynamic communication topology wherein communication events are triggered via state-dependent resettings inspired by thermodynamic phase transition driving parameters from condensed matter physics [12]. Due to space limitations we omit all proofs in the paper; the detailed proofs of all the results appear in [13].

2 A HYBRID THERMODYNAMIC CONSENSUS CONTROL ARCHITECTURE

To present the key ideas for developing a thermodynamic-based hybrid control architecture for information consensus consider the hybrid dynamical system $\mathcal{G}$

$$\dot{x}(t) = f_{\rm c}(x(t)), \qquad x(0) = x_0, \qquad x(t) \notin \mathcal{Z}, \tag{1}$$

$$\Delta x(t) = f_{\rm d}(x(t)), \qquad x(t) \in \mathcal{Z}, \tag{2}$$

where, for every $t \geq 0$, $x(t) \in \mathscr{D} \subseteq \mathbb{R}^n$, $\mathscr{D}$ is an open set with $0 \in \mathscr{D}$, $\Delta x(t) \triangleq x(t^+) - x(t)$, where $x(t^+) \triangleq x(t) + f_{\rm d}(x(t)) = \lim_{\varepsilon \to 0^+} x(t+\varepsilon)$, $f_{\rm c} : \mathscr{D} \to \mathbb{R}^n$ is Lipschitz continuous and satisfies $f_{\rm c}(0) = 0$, $f_{\rm d} : \mathscr{D} \to \mathbb{R}^n$ is continuous, and $\mathscr{Z} \subset \mathscr{D}$ is the *resetting set*. Note that $x_{\rm e} \in \mathscr{D}$ is an equilibrium point of (1) and (2) if and only if $f_{\rm c}(x_{\rm e}) = 0$ and $f_{\rm d}(x_{\rm e}) = 0$. We refer to the differential equation (1) as the *continuous-time dynamics*, and we refer to the difference equation (2) as the *resetting law*. For a discussion on solutions to impulsive differential equations, see [9].

Next, we develop a thermodynamically motivated information consensus framework for multiagent nonlinear systems to achieve semistability and state equipartition. The consensus problem we consider involves a dynamic communication graph with intermittent information over the dynamical network characterized by the multiagent impulsive dynamical systems $\mathscr{G}_i$ given by

$$\dot{x}_i(t) = u_{{\rm c}i}(t), \quad x_i(0) = x_{i0}, \quad x_i(t) \notin \mathscr{Z}_i, \tag{3}$$

$$\Delta x_i(t) = u_{{\rm d}i}(t), \quad x_i(t) \in \mathscr{Z}_i, \quad i = 1, \ldots, q, \tag{4}$$

where, for every $t \geq 0$, $x_i(t) \in \mathbb{R}$ denotes the information state and $u_{{\rm c}i}(t)$ and $u_{{\rm d}i}(t) \in \mathbb{R}$, respectively, denote the continuous and discrete information control inputs associated with the local resetting set $\mathscr{Z}_i \subset \mathbb{R}$, $i \in \{1, \ldots, q\}$.

The hybrid consensus protocol is given by

$$u_{{\rm c}i}(t) = \sum_{j=1, i\neq j}^{q} \phi_{{\rm c}ij}(x_i(t), x_j(t)), \tag{5}$$

$$u_{{\rm d}i}(t) = \sum_{j=1, i\neq j}^{q} \phi_{{\rm d}ij}(x_i(t), x_j(t)), \tag{6}$$

where, for all $i, j = 1, \ldots, q$, $i \neq j$, $\phi_{{\rm c}ij}(\cdot,\cdot)$ is locally Lipschitz continuous, $\phi_{{\rm d}ij}(\cdot,\cdot)$ is continuous, $\phi_{{\rm c}ij}(x_i, x_j) = -\phi_{{\rm c}ji}(x_j, x_i)$, and $\phi_{{\rm d}ij}(x_i, x_j) = -\phi_{{\rm d}ji}(x_j, x_i)$. In this case, the closed-loop system (3)–(6) is given by

$$\dot{x}_i(t) = \sum_{j=1, i\neq j}^{q} \phi_{{\rm c}ij}(x_i(t), x_j(t)), \quad x_i(0) = x_{i0}, \qquad x_i(t) \notin \mathscr{Z}_i, \quad i = 1, \ldots, q, \tag{7}$$

$$\Delta x_i(t) = \sum_{j=1, i\neq j}^{q} \phi_{{\rm d}ij}(x_i(t), x_j(t)), \quad x_i(t) \in \mathscr{Z}_i, \tag{8}$$

or, equivalently, in vector form given by (1) and (2), where $x(t) \triangleq [x_1(t), \ldots, x_q(t)]^{\rm T} \in \mathbb{R}^q$, $f_{\rm c}(x(t)) \triangleq [f_{{\rm c}1}(x(t)), \ldots, f_{{\rm c}q}(x(t))]^{\rm T} \in \mathbb{R}^q$, $f_{\rm d}(x(t)) \triangleq [f_{{\rm d}1}(x(t)), \ldots, f_{{\rm d}q}(x(t))]^{\rm T} \in \mathbb{R}^q$, and $\mathscr{Z} \triangleq \cup_{i=1}^{q} \{x \in \mathbb{R}^q : x_i \in \mathscr{Z}_i\}$, with, for $i, j = 1, \ldots, q$,

$$f_{{\rm c}i}(x(t)) = \sum_{j=1, i\neq j}^{q} \phi_{{\rm c}ij}(x_i(t), x_j(t)), \tag{9}$$

$$f_{{\rm d}i}(x(t)) = \sum_{j=1, i\neq j}^{q} \phi_{{\rm d}ij}(x_i(t), x_j(t)). \tag{10}$$

Note that $\mathscr{G}$ given by (1) and (2) describe an interconnected network where information states are updated using a distributed hybrid controller involving neighbor-to-neighbor interaction between agents. Furthermore, this hybrid control protocol involves a design procedure for consensus with intermittent transmission of information as defined by the local resetting sets $\mathscr{Z}_i$, $i \in \{1, \ldots, q\}$.

In order to define the global resetting set $\mathscr{Z}$ in terms of the local resetting sets $\mathscr{Z}_i$, $i = 1, \ldots, q$, associated with $\mathscr{G}$, we require some additional notation. Let $\mathscr{O}_i$ denote the set of all agents with information flowing out to the ith agent and let $\mathscr{I}_i$ denote the set of all agents receiving information from the ith agent. We define the local resetting sets $\mathscr{Z}_i$ by

$$\mathscr{Z}_i \triangleq \Big\{ x_i \in \mathbb{R} : \sum_{j \in \mathscr{O}_i} \phi_{{\rm c}ij}(x_i, x_j)(x_i - x_j) - \sum_{j \in \mathscr{I}_i} \phi_{{\rm c}ij}(x_i, x_j)(x_i - x_j) = 0, \text{ and } x_i \neq x_j,\ j \in \mathscr{O}_i \cup \mathscr{I}_i \Big\}, \quad i = 1, \ldots, q, \tag{11}$$

with $\mathscr{Z} \triangleq \bigcup_{i=1}^{q} \{x \in \mathbb{R}^q : x_i \in \mathscr{Z}_i\}$. The resetting set (11) is proposed in [12] and, as noted in Remark 2.1, is consistent with thermodynamic principles.

To ensure a thermodynamically consistent information flow model, we make the following assumptions on the information flow functions $\phi_{{\rm c}ij}(\cdot,\cdot)$, $i, j = 1, \ldots, q$, between state resettings:

Assumption 2.1. *The connectivity matrix $\mathscr{C} \in \mathbb{R}^{q \times q}$ associated with the hybrid multiagent dynamical system $\mathscr{G}$ given by* (1) *and* (2) *is defined by*

$$\mathscr{C}_{(i,j)} = \begin{cases} 0, & \text{if } \phi_{{\rm c}ij}(x_i(t), x_j(t)) \equiv 0, \\ 1, & \text{otherwise}, \end{cases} \quad i \neq j, \quad i, j = 1, \ldots, q, \quad t \geq 0, \tag{12}$$

$$\mathscr{C}_{(i,i)} = -\sum_{k=1, k\neq i}^{q} \mathscr{C}_{(k,i)}, \quad i = j, \quad i = 1, \ldots, q, \tag{13}$$

with rank $\mathscr{C} = q-1$, *and for* $\mathscr{C}_{(i,j)} = 1$, $i \neq j$, $\phi_{{\rm c}ij}(x_i(t), x_j(t)) = 0$ *if and only if* $x_i(t) = x_j(t)$ *for all* $x(t) \notin \mathscr{Z}$, $t \geq 0$.

Assumption 2.2. *For* $i, j = 1, \ldots, q$, $i \neq j$, $[x_i(t) - x_j(t)] \cdot \phi_{{\rm c}ij}(x_i(t), x_j(t)) \leq 0$, $x(t) \notin \mathscr{Z}$, $t \geq 0$.

Furthermore, across resettings the information difference must satisfy the following assumption:

Assumption 2.3. *For* $i, j = 1, \ldots, q$, $[x_i(t_{k+1}) - x_j(t_{k+1})] \cdot [x_i(t_k) - x_j(t_k)] \geq 0$ *for all* $x_i(t_k) \neq x_j(t_k)$, $x(t_k) \in \mathscr{Z}$, $k \in \mathbb{Z}_+$.

The condition $\phi_{{\rm c}ij}(x_i(t), x_j(t)) = 0$ if and only if $x_i(t) = x_j(t)$, $i \neq j$, for all $x(t) \notin \mathscr{Z}$ implies that agents $\mathscr{G}_i$ and $\mathscr{G}_j$ are *connected*, and hence, can share information; alternatively $\phi_{{\rm c}ij}(x_i(t), x_j(t)) \equiv 0$ implies that agents $\mathscr{G}_i$ and $\mathscr{G}_j$ are *disconnected*, and hence, cannot share information.

Assumption 2.1 implies that if the energies or information in the connected agents $\mathcal{G}_i$ and $\mathcal{G}_j$ are equal, then energy or information exchange between these agents is not possible. This statement is reminiscent of the *zeroth law of thermodynamics*, which postulates that temperature equality is a necessary and sufficient condition for thermal equilibrium. Furthermore, if $\mathcal{C} = \mathcal{C}^{\mathrm{T}}$ and rank $\mathcal{C} = q-1$, then it follows that the connectivity matrix $\mathcal{C}$ is irreducible, which implies that for any pair of agents $\mathcal{G}_i$ and $\mathcal{G}_j$, $i \neq j$, of $\mathcal{G}$ there exists a sequence of information connectors (information arcs) of $\mathcal{G}$ that connect agents $\mathcal{G}_i$ and $\mathcal{G}_j$.

Assumption 2.2 implies that energy or information flows from more energetic or information rich agents to less energetic or information poor agents and is reminiscent of the *second law of thermodynamics*, which states that heat (i.e., energy in transition) must flow in the direction of lower temperatures. Finally, Assumption 2.3 implies that for any pair of connected agents $\mathcal{G}_i$ and $\mathcal{G}_j$, $i \neq j$, the energy or information difference between consecutive jumps is monotonic.

The following definition for semistability is needed for the main result of the paper. Recall that for addressing the stability of an impulsive dynamical system the usual stability definitions are valid [9]. For the statement of the next definition and theorem, $\mathcal{B}_\delta(x)$ denotes the open ball centered at x with radius δ and $\mathbf{e} \in \mathbb{R}^q$ denotes the ones vector of order q, that is, $\mathbf{e} = [1,\ldots,q]^{\mathrm{T}}$.

Definition 2.1. *An equilibrium solution $x(t) \equiv x_{\mathrm{e}} \in \mathbb{R}^n$ to (1) and (2) is* semistable *if it is Lyapunov stable and there exists $\delta > 0$ such that if $x_0 \in \mathcal{B}_\delta(x_{\mathrm{e}})$, then $\lim_{t\to\infty} x(t)$ exists and corresponds to a Lyapunov stable equilibrium point.*

Theorem 2.1. *Consider the closed-loop hybrid multiagent dynamical system $\mathcal{G}$ given by (1) and (2) with resetting set $\mathcal{Z}$ given by (11), and assume Assumptions 2.1, 2.2, and 2.3 hold. Then, for every $\alpha \geq 0$, $\alpha\mathbf{e}$ is a semistable equilibrium state of $\mathcal{G}$. Furthermore, $x(t) \to \frac{1}{q}\mathbf{e}\mathbf{e}^{\mathrm{T}}x(0)$ as $t \to \infty$ and $\frac{1}{q}\mathbf{e}\mathbf{e}^{\mathrm{T}}x(0)$ is a semistable equilibrium state.*

Next, we provide explicit connections of the proposed thermodynamic-based consensus control architecture with the recently developed notion of hybrid thermodynamics [12].

Definition 2.2. *For the distributed hybrid consensus control protocol $\mathcal{G}$ given by (1) and (2), a function $\mathcal{S} : \mathbb{R}^q \to \mathbb{R}$ satisfying*

$$\mathcal{S}(x(T)) \geq \mathcal{S}(x(t_1)), \quad t_1 \leq t_k < T, \quad k \in \mathbb{Z}_+, \tag{14}$$

is called an entropy *function of $\mathcal{G}$.*

The next result gives necessary and sufficient conditions for establishing the existence of a hybrid entropy function of $\mathcal{G}$ over an interval $t \in (t_k, t_{k+1}]$ involving the consecutive resetting times t_k and t_{k+1}, $k \in \mathbb{Z}_+$.

Theorem 2.2. *Consider the closed-loop hybrid multiagent dynamical system $\mathcal{G}$ given by (1) and (2), and assume Assumptions 2.1, 2.2, and 2.3 hold. Then a function $\mathcal{S} : \mathbb{R}^q \to \mathbb{R}$ is an entropy function of $\mathcal{G}$ if and only if*

$$\mathcal{S}(x(\hat{t})) \geq \mathcal{S}(x(t)), \quad t_k < t \leq \hat{t} \leq t_{k+1}, \tag{15}$$

$$\mathcal{S}(x(t_k^+)) \geq \mathcal{S}(x(t_k)), \quad k \in \mathbb{Z}_+. \tag{16}$$

The next theorem establishes the existence of a continuously differentiable entropy function for the closed-loop hybrid multiagent dynamical system $\mathcal{G}$ given by (1) and (2).

Theorem 2.3. *Consider the closed-loop hybrid multiagent dynamical system $\mathcal{G}$ given by (1) and (2), and assume Assumptions 2.2 and 2.3 hold. Then the function $\mathcal{S} : \mathbb{R}^q \to \mathbb{R}$ given by*

$$\mathcal{S}(x) = \mathbf{e}^{\mathrm{T}}\mathbf{log}_e(c\mathbf{e}+x) - q\log_e c, \tag{17}$$

where $\mathbf{log}_e(c\mathbf{e}+x)$ denotes the vector natural logarithm given by $[\log_e(c+x_1),\ldots,\log_e(c+x_q)]^{\mathrm{T}}$ and $c > \|x\|_\infty$, is a continuously differentiable entropy function of $\mathcal{G}$. In addition,

$$\dot{\mathcal{S}}(x(t)) \geq 0, \quad x(t) \notin \mathcal{Z}, \quad t_k < t < t_{k+1}, \tag{18}$$

$$\Delta\mathcal{S}(x(t_k)) \geq 0, \quad x(t_k) \in \mathcal{Z}, \quad k \in \mathbb{Z}_+. \tag{19}$$

Remark 2.1. *It follows from the proof of Theorem 2.3 that* (11) *implies that if the time rate of change of the difference in the input information flow and output information flow between any pair of connected agent entropies is zero and consensus is not reached, then a resetting occurs. For details, see [13].*

It follows from Theorems 2.2 and 2.3 that the entropy function (17) satisfies (14) as an equality for an equilibrium (equipartitioned) process and as a strict inequality for a nonequilibrium (nonequipartitioned) process. The entropy expression given by (17) is identical in form to the Boltzmann entropy for statistical thermodynamics [8]. In addition, $\mathcal{S}(x)$ given by (17) achieves a maximum when all the information states x_i, $i = 1,\ldots,q$, are equal [7, 8]. Inequality (14) is a generalization of Clausius' inequality for equilibrium and nonequilibrium thermodynamics as well as reversible and irreversible thermodynamics as applied to adiabatically isolated hybrid thermodynamic systems involving discontinuous phase transitions. For details, see [12].

3 ILLUSTRATIVE NUMERICAL EXAMPLE

In this section, we demonstrate the proposed distributed hybrid consensus framework on a set of aircraft achieving pitch rate consensus. Specifically, consider the multiagent system comprised of the controlled longitudinal motion of seven Boeing 747 airplanes linearized at an altitude of 40 kft and a velocity of 774 ft/sec given by

$$\dot{z}_i(t) = Az_i(t) + B\delta_i(t), \quad z_i(0) = z_{i_0}, \quad i = 1,\ldots,7, \tag{20}$$

where $z_i(t) = [v_{x_i}(t), v_{z_i}(t), q_i(t), \theta_{e_i}(t)]^{\mathrm{T}} \in \mathbb{R}^4$, is state vector of aircraft with $v_{x_i}(t)$, $t \geq 0$, representing the x–body–axis component of the velocity of the airplane center of mass with respect to the reference axes (in ft/sec), $v_{z_i}(t)$, $t \geq 0$, representing the z–body–axis component of the velocity of the airplane center of mass with respect to the reference axes (in ft/sec), $q_i(t)$, $t \geq 0$, representing the y–body–axis component of the angular velocity of the airplane (pitch rate) with respect to the reference axes (in crad/sec), $\theta_{e_i}(t)$, $t \geq 0$, representing the pitch Euler angle of the

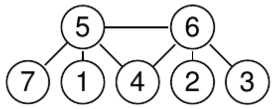

FIGURE 1. Aircraft communication topology.

airplane body axes with respect to the reference axes (in crad), $\delta(t)$, $t \geq 0$, representing the elevator control input (in crad), and A and B are the plant and control matrices [13].

We propose a two-level control hierarchy composed of a lower-level controller for command following and a higher-level hybrid consensus controller for pitch rate consensus with a communication topology as shown in Figure 1. To address the lower-level controller design, let $x_i(t)$, $i = 1,\ldots,7$, $t \geq 0$, denote an information command generated by (7) and (8) (i.e., the guidance command) and let $s_i(t)$, $i = 1,\ldots,7$, $t \geq 0$, denote the integrator state satisfying $\dot{s}_i(t) = Ez_i(t) - x_i(t)$, $i = 1,\ldots,7$, where $E = [0,0,1,0]$. Now, defining the augmented state $\hat{z}_i(t) \triangleq [z_i^{\mathrm{T}}(t), s_i(t)]^{\mathrm{T}} \in \mathbb{R}^5$, we obtain

$$\dot{\hat{z}}_i(t) = \begin{bmatrix} A & 0 \\ E & 0 \end{bmatrix} \hat{z}_i(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} \delta_i(t) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} x_i(t), \quad \hat{z}_i(0) = \hat{z}_{i0}.$$

Furthermore, let the elevator control input be given by $\delta_i(t) = -K\hat{z}_i(t)$, $i = 1,\ldots,7$, which is designed based on an optimal linear-quadratic regulator.

For the higher-level hybrid consensus controller design, we use (7) with functions $\phi_{\mathrm{c}ij}(x_i(t),x_j(t)) = (x_j(t) - x_i(t))^{\frac{1}{3}}$, $i,j \in \{1,\ldots,7\}$, $i \neq j$, and (8) with $\phi_{\mathrm{d}ij}(x_i(t),x_j(t)) = (x_j(t) - x_i(t))/3$, $i,j \in \{1,\ldots,7\}$, $i \neq j$, to generate the information state $x(t)$, $t \geq 0$, that has a direct effect on the lower-level controller design to achieve pitch rate consensus. Here, we used the resetting set $\mathscr{Z}_i$ given by (11). Figure 2 shows the information command signals and pitch rate of each aircraft versus time for the proposed hybrid thermodynamic control protocol.

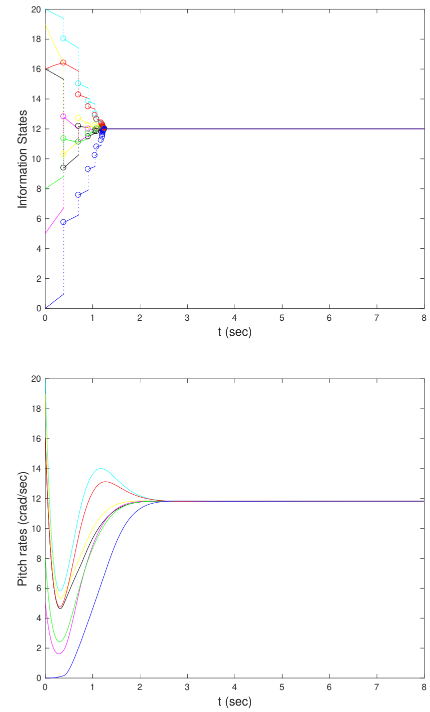

FIGURE 2. Closed-loop information command signal $x_i(t)$ (top) and pitch rate $q_i(t)$ (bottom) trajectories with the proposed higher-level hybrid consensus protocol.

4 CONCLUSION

In this paper, we presented a thermodynamically-based framework for addressing consensus problems for hybrid multiagent dynamical systems with bidirectional communication between agents in the network. Specifically, hybrid nonlinear network protocols were designed that guarantee convergence to Lyapunov stable equilibria. Our analysis relies on several tools from algebraic graph theory, semistability, impulsive differential equations, and hybrid dynamical thermodynamics [8,12]. Future research will explore extending the proposed framework to include directed communication topologies as well as developing hybrid information consensus algorithms for achieving coordination tasks in finite time.

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