# A Non-Equilibrium Mathematical Economic Framework on the basis of Non-Equilibrium Thermodynamics

## Preprint


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# A NON-EQUILIBRIUM MATHEMATICAL ECONOMIC FRAMEWORK ON THE BASIS OF NON-EQUILIBRIUM THERMODYNAMICS

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## ABSTRACT

*A sketch of a mathematical economic framework that does not rely on the false assumptions of equilibrium, rationality and utility-maximisation is presented – on the foundation of the math and approach used in non-equilibrium thermodynamics. In addition, the question of whether there is a need for a mathematical framework that does not assume specific morals is addressed as well as the importance of non-reductionist approach to non-linear complex systems.*

Keywords: non-equilibrium economics, complex systems, dynamic systems, non-linear equations, non-equilibrium thermodynamics, irreversibility

## 1. INTRODUCTION

"You don't have to choose between being compassionate and being scientific – so go and do both, good luck!" – Robert Sapolsky [1]

"We have reason to hope for a future economic science that is more parsimonious, conceptually clearer and less subjective. It will resemble reality more closely and be better aligned with our moral intuitions." – Ole Peters [2]

As economics is both a social and non-social science, appropriate economic theories shouldn't choose between the two sides but should include both parts. The simple mathematical economic framework presented in this extended abstract fulfills this expectation, it allows for the inclusion of various moral, social approaches, while describing the inherently irreversible and non-linear nature of economic processes and including conservation of matter.

The mathematical economic framework presented below relies on the concepts and advances of non-equilibrium thermodynamics, as although there is no emphasis on irreversibility in the most common mathematical economic frameworks, irreversibility plays a vital role in thermodynamics [3]. While there are crucial differences between physical and economic systems that have to be considered [4], the correspondence that exists may allow the formulation of economic and social models from an irreversible perspective [3].

## 2. ON THE FUNDAMENTAL NEED FOR A MATHEMATICAL ECONOMIC FRAMEWORK THAT DOES NOT ASSUME SPECIFIC MORALS

That the most widely taught and relied on mathematical economic theory does not describe real-world phenomena is well-known [5-9]. In the words of Bernard Beaudreau [10]: "It goes without saying that the very core field in modern economics has a questionable track record, scientifically speaking. While it is elegant in its axioms and construction, logical in its reasoning and exhaustive in its breadth, it has been less than successful where it counts, namely shedding light on real-world phenomena".

Nevertheless, there is a prevailing skepticism about whether other foundations are possible and would give results that are more comparable to the real-world phenomena. In addition, the elaborated theoretical ecosystem built on the basis of the neoclassical economic theory makes 'entering the market' expensive, thereby limiting the possibilities of choice and leading to substantial effort spent on mending the neoclassical economic theory and thus further increasing its ecosystem.

In the following paragraphs the possibility and need for replacing instead of mending the neoclassical economic framework [11] will be argued for. The argument in a nutshell is, that neoclassical economic theory has a detrimental effect on our society, not only due to that the decisions based on it are far from being optimal, but also because it spreads a negative world-view and by that encourages a negative self-fulfilling prophecy.

### 2.1 Assumptions of the neoclassical economic theory

The neoclassical economic theory relies on several unrealistic assumptions, in this paper three of them will be pinpointed to and quickly analyzed, also known as Solow's trinity [12]: equilibrium, rationality and utility maximisation (a.k.a. greediness).

The assumption of equilibrium enables the undisprovable claim that in an ideal, equilibrium situation the theory would work. The assumption of rationality puts the responsibility of failure on people, by overlooking that appropriate resources for making rational choices are not available for any human being. The assumption of greediness might be more debatable, however, there is psychological evidence that people adapt to the assumptions about them, thus just by assuming greediness, people will be more greedy than without it – in addition to normalising and presenting this property detrimental for society as a positive one. As a consequence, the assumptions of neoclassical economic theory are not only false but function as nocebos and have detrimental effects on our society [13], and might lead to Herbet Marcuse's empty prosperity [14].

### 2.2 Realistic assumptions about human behaviour

That the reality about the human species is the direct opposite of the assumptions of neoclassical economic theory is getting increasing support in the research community. It is becoming widely accepted that our species is evolutionarily successful exactly because there is a strong moral dimension to human social cooperation and collaboration [13, 15] and that the task of the economy is to provide for the sustaining and flourishing of life and not to master life [9, 16].

### 2.3 Neoclassical economic theory and the material world

As John Sterman pointed out [17], "The most important assumptions of a model are not in the equations, but what's not in them; not in the documentation, but unstated; not in the variables on the computer screen, but in the blank spaces around them." and that is even more prevalent in the case of neoclassical economic theory.

Neoclassical economic theory in its basic form lacks the acknowledgment that we live in a material world thus economic processes depend on the available energy and materials, the society where it takes place [9].

A step towards not including the material world in the equations of economics was taken by John Stuart Mill, whose aim was to make a clear distinction between natural and social sciences, so that the latter should solely focus on the laws of mind [9]. A further step in this path was Samuelson's flow-diagram, that was supposed to represent only part of the society, but as it was understood as a general representation of the economic processes, it taught highly simplified concepts to millions of students [9].

## 3. ON THE IMPORTANCE OF A NON-REDUCTIONIST APPROACH

As Robert May pinned it in 1976 [18]: "Not only in research, but also in the everyday world of politics and economics, we would all be better off if more people realized that simple nonlinear systems do not necessarily possess simple dynamical properties." Describing nonlinear, complex systems in a reductionist way introduces simplifications that change the core properties of the very system we are investigating [19].

Thus even the most basic economic model has to include nonlinearity, as economic systems are inherently dynamic systems, and especially when accounting for external factors, like innovation, natural catastrophes or other disruptions [20]. Therefore our model presented in the next part reflects this complexity, while conserving simple and easy-to-interpret equations.

## 4. SKETCH OF THE PROPOSED MATHEMATICAL FRAMEWORK

The mathematical framework presented below is based on the description presented in the book of R. U. Ayres and K. Martinás: *On the reappraisal of microeconomics* [21]. It is important to note, that it relies on several assumptions that do not always hold true and have to be taken into account. The main differences between these assumptions and those of the neoclassical economic theory are that they are explicitly worded, more realistic and less restricting.

1. Instead of utility: Production and trading level is proportional to the expected profit.
2. Instead of rationality: The purpose of economic action is to increase the expected economic welfare, or in other words, to avoid avoidable losses. That is in line with Herbert Simon's satisficing rule [22].
3. Instead of equilibrium: Evolution of an economic system is described by the balance equation for stocks of material goods and money.

### 4.1 Welfare function

The welfare function (Z) of an economic agent (α) is defined, as a function of the stocks ($X_i$ where i: 1… n) of goods and money (M) belonging to the economic agent:

$$Z_\alpha(t) = Z_\alpha(X(1...n, t), M(t)) \qquad (1)$$

Sign convention is selected so that $\Delta Z > 0$ for allowed (no-loss) processes, and $\Delta Z < 0$ for forbidden (loss-making) transactions, (t) refers to time-dependence.

The welfare function (Z) is not simply the sum of the stocks, as welfare is a subjective term, its dependence on the stocks is defined by the agent and is time-dependent (e.g. the agent can learn).

In addition to the welfare function, we introduce $h_i$: coefficient for certain independent material goods. $h_i$ translates a certain change in the quantity of that specific good to the change in the internal value for a certain agent ($V_{i,\alpha}$).

$$h_i \Delta X_i = \Delta V_{i,\alpha} \qquad (2)$$

This internal value is dependent on the specific state of the agent, thus time and other possessions.

We also introduce $h_M$: a coefficient that translates money to internal value. $h_M$ translates a certain change in the quantity of money to the change in the internal value for a certain agent ($V_{M,\alpha}$).

$$h_M \Delta M = \Delta V_{M,\alpha} \qquad (3)$$

The expected economic welfare of the agent is:

$$Z(t) = w(t) (\Sigma_i V_i (t) + V_M (t)) \quad (4)$$

Thus, the expected change in the economic welfare of the agent is:

$$\Delta Z(t) = w(t) (\Sigma_i \Delta V_i (t) + \Delta V_M (t)) \quad (5)$$

Where $\Delta Z$ is the change in Z, $w_M$ is the change of welfare dependent on value, $\Delta V_i$ is the change in the value of stocks represented in internal monetary terms and $\Delta V_M$ is the change in the value of the stocked money.

### 4.2 Production and consumption

Following the same logic, the expected gains and losses in case of production are as follows:

$$\Delta G^C(t) = \Sigma h_i(t)\Delta X^C_i(t) \quad (6)$$

Where $\Delta G^C$ is the gain from consumption-production, $\Delta X^C_i$ is the change of the $i^{th}$ good in a unit consumption-production. It is positive for the products and by-products, and negative for the input.

The technology defines what can be produced from which inputs in a unit of time, thus it defines the production vector, $X^C_i$, for any i good. The agent defines the production level. Technology and capital gives an upper limit (maximal yield ($y_{max}$)), but real systems work with less efficiency.

Then an equation for the change of welfare function due to production can be formalised as follows:

$$\Delta Z^C(t) = w_M(t)\Delta G^C(t) \quad (7)$$

Based on our assumption 1, the production level is proportional to the expected profit, thus we can write the following equation:

$$\Sigma y_i(t) = \Sigma L^C_i(t)h_i(t)\Delta X^C_i(t) \quad (8)$$

Where $\Sigma y_i$ is the created goods, $L^C_i$ is the parameter describing the proportionality.

### 4.3 Trade

The expected gain (profit) in trade of a unit of a material good i for price $p_i$ is:

$$G^T_\alpha = \Sigma(V_{\alpha,i} - p_i) \quad (9)$$

Then the equation for the change of welfare function due to trading:

$$\Delta Z^T(t) = w_M(t)\Delta G^T(t) \quad (10)$$

When talking about trade, traded quantity when agent α trades with the agent β at price $p_i$ will be proportional to the expected gain, that is

$$y_{\alpha,i} = L^T_{\alpha,i} (V_{\alpha,i} - p_i) \quad (11)$$

Trade is viable only if the agent β agrees to the same quantity with opposite sign, thus:

$$y_{\beta,i} = L^T_{\beta,i} (V_{\beta,i} - p_i) = - y_{\alpha,i} \quad (12)$$

We separate trade from the consumption-production processes, therefore during trade no new goods are created for the agents participating in the bargaining process:

$$\Sigma_{\alpha\beta}\, y_{\alpha\beta,i} = \Sigma_{\alpha\beta}\, L^T_{\alpha\beta,i} (V_{\alpha\beta,i} - p_i) = 0 \quad (13)$$

Thus, the sum of all traded material goods is zero, however the sum of gains does not have to be, and based on assumption 2 it is always positive.

The sum of gains is:

$$\Delta G^T = V_{\alpha,i} - p_i - (V_{\beta,i} - p_i) = V_{\alpha,i} - V_{\beta,i} \quad (14)$$

Based on our assumption 2 $\Delta Z_T$ has to be positive for both agents, $\Delta G_T$ likewise. Therefore, if $V_{\alpha,i} < V_{\beta,i}$, agent β will buy from agent α, in the opposite case the inverse will be true. However, in case one of the “participants” in the trade is nature or an agent without the possibility to make a free choice, that assumption does not hold.

### 4.4 The mathematical framework

Based on the previous paragraphs, three coupled equations can be formulated:

$$X_\alpha(t + 1) = X_\alpha(t)+ L^T_\alpha(V_\alpha - p) + L^C_\alpha V^C_\alpha \quad (15)$$

Meaning that the stocks of goods and money at time t+1 equals to the stocks of goods and money at time t plus the results of the trading processes and the expected profit times the produced goods.

$$M_\alpha(t + 1) = M_\alpha(t) - pL^T_\alpha(V_\alpha - p) \quad (16)$$

Meaning that the amount of money owned by the agent at time t+1 equals to the amount of money owned by them at time t, minus the money spent or plus the money gained during the trading process.

$$y^T = L^T(V- p) = 0 \quad (17)$$

Meaning that we have conservation of matter during trading.

We have three simple, easy-to-interpret coupled nonlinear equations. They represent a complex system, and thus the solutions and results are not straightforward and can not be calculated linearly: they need input-information and programming.

### 4.5 Calculations based on the mathematical framework

Some sample calculations were performed and analyzed e.g. in the book of R. U. Ayres and K. Martinás: *On the reappraisal of microeconomics* [21]. Here one of them will be quickly summarized and discussed.

In the book a simple system of 3 agents, 3 goods and money were simulated and the results discussed. The system went through several phases: order out of chaos, stability and then instability on the long-term. However, by introducing a new rule of taxing 10% of the wealth difference between the richest agent and the poorest, and reallocating it to the poorest after each cycle, the long-term instability was prevented and the system remained stable for the length of the simulation [21].

This example shows one of the advantages of the model presented in this extended abstract: despite its simplicity, it describes non-linear complex systems, like our economy.

## 5. CONCLUSION

The sketch of a mathematical economic model presented here does not rely on the false assumptions of equilibrium, rationality and utility-maximisation, its assumptions are explicitly stated and accommodate a wide variety of needs and ethics. Furthermore, the model depicts the inherent complexity of the economic system while its basic assumptions and concepts are clear and concise.

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